Solution for #67

Limits will never go away!

$$f(x)=\begin{cases} 1 & x=0 \\ ax+b & 0<x\le 1 \\ x^2+4x+c & 1\le x\le c \end{cases}$$

Work on both continuity and differentiability

$f(0) = \lim_{x\to 0} f(x)$ $\quad f(0)=1$

$\lim_{x\to 0^+} f(x) = b$ $\quad$ Hence $b=1$

need $f(1) = \lim_{x\to 1} f(x)$

$\lim_{x\to 1^-} f(x) = a+1$ $\quad \lim_{x\to 1^+} f(x) = 5+c$

need $a+1 = 5+c$

$\lim_{x\to 1^-} f'(x) = a$ $\quad \lim_{x\to 1^+} f'(x) = 6$

Hence $a=6$, $b=1$

Now find c

$a+1 = 5+c$ $\quad$ Hence $c=2$

$a+1 = 5+c$