Definition of a Definite Integral

$$\lim_{dx \to 0} \sum_{i=1}^{n} f(c_i)dx = \int_a^b f(x)dx$$

What?!!!
Think of the rectangles! We are summing up the area of numerous rectangles and we want the width of the rectangles $[dx]$to get close to zero and the number of rectangles, n, to get close to infinity.

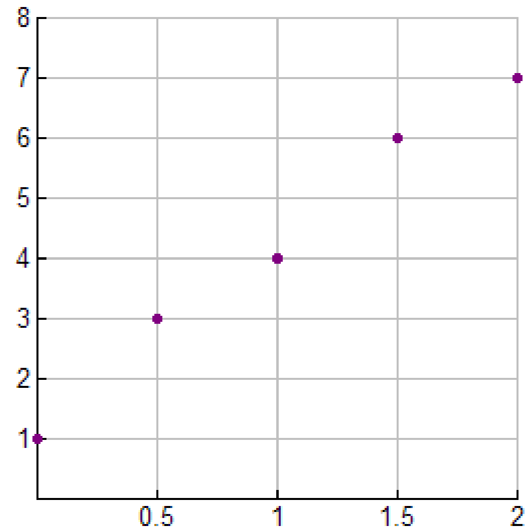
♪ If f is continuous on $[a, b]$, then f can be integrated on $[a, b]$.

Today we are going to concentrate on finding definite integrals using one of the RAM.

Let's consider the following problem
The table below contains values of a continuous function f at several inputs x.

x	0	0.5	1	1.5	2
$f(x)$	1	3	4	6	7

Estimate $\int_0^2 f(x)\,dx$ using a right-hand sum [RRAM] with four equal subintervals, and draw a sketch that illustrates this sum geometrically.



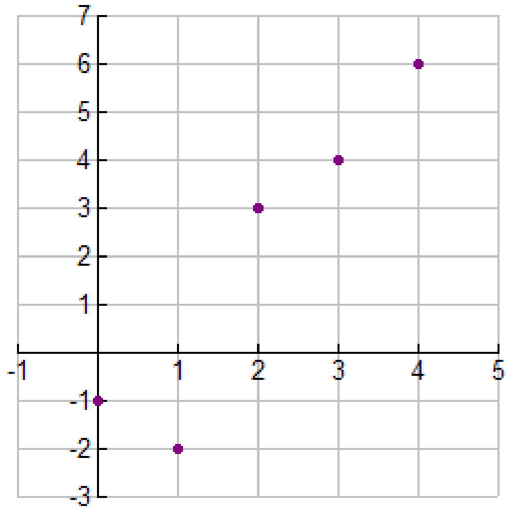
$$\int_0^2 f(x)\,dx \approx RRAM$$

$$RRAM = 0.5\left[f(0.5) + f(1) + f(1.5) + f(2)\right]$$
$$RRAM = 0.5[3 + 4 + 6 + 7]$$
$$RRAM = 0.5[20]$$
$$RRAM = 10$$

We are simply adding up the area of the four rectangles! We need to be careful whenever our $f(x)$ values are negative.

What if we had the following values for $g(x)$ and we want to find $\int_0^4 g(x)\,dx$ using a left-hand sum with four equal subintervals?

x	0	1	2	3	4
$g(x)$	-1	-2	3	4	6



$$\int_0^4 g(x)\ dx \approx \text{LRAM}$$

$$LRAM = (1)\left[g(0) + g(1) + g(2) + g(3)\right]$$
$$LRAM = (1)\left[-1 + -2 + 3 + 4\right]$$

$$LRAM = 4$$

Now let's sing our Riemann Sum Song!

Riemann Sums
(sung to the tune of Jingle Bells)

Riemann Sums, Riemann Sums
Counting Areas
Of rectangles whose widths get small
We need to count them all
Riemann Sums, Riemann Sums
Counting Areas
Of rectangles whose widths get small
We need to count them all.

We learn to integrate
It's really lots of fun.
It's easier to find
Than those old Riemann Sums
We learn to sub a u
When things get sort of hard
But most of all we tabulate
When we get sick of parts.
[repeat the refrain]

Let's do some more AP Problems!

I can't post the questions but I can put a link to them.
http://www.collegeboard.com/prod_downloads/student/testing/ap/calculus_ab/ap07_calculus_ab_frq.pdf

2007#5

$$\int_0^{12} r'(t)\,dt \approx \text{RRAM}$$

$$\text{RRAM} = (2)(r'(2)) + (3)(r'(5)) + (2)(r'(7)) + (4)(r'(11)) + (1)(r'(12))$$

$$= 19.3 \text{ ft}$$

$\int_0^{12} r'(t)\,dt$ gives us the CHANGE in the RADIUS IN FT FROM $t = 0$ TO $t = 12$.

http://www.collegeboard.com/prod_downloads/student/testing/ap/calculus_ab/ap06_frq_calculus%20ab.pdf
2006 AB4

$$\int_{10}^{70} v(t)\,dt \approx \text{MRAM}$$

$$\text{MRAM} = 20\left[v(20) + v(40) + v(60)\right]$$

$$= 2020 \text{ ft}$$

$\int_{10}^{70} v(t)\,dt$ gives us distance in ft, traveled by rocket from T = 10 to T = 70 seconds

http://www.collegeboard.com/prod_downloads/ap/students/calculus/ap05_frq_calculus_ab.pdf
2005 AB5

$$\int_0^{24} v(t)\,dt = \frac{b_1 + b_2}{2}(h)$$
$$= \frac{24+12}{2}(20)$$
$$= 360\text{m}$$

$\int_0^{24} v(t)\,dt$ gives us the distance traveled by Speed Racer in m, from t = 0 to t = 24 sec

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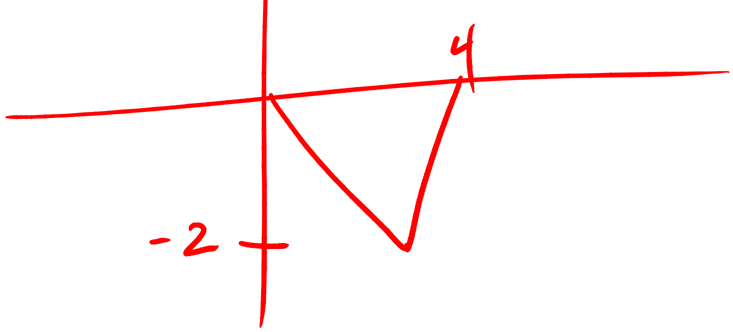
47 a-e



$$\int_0^4 f(x)\,dx$$
$$= -\left(\tfrac{1}{2}bh\right)$$
$$= -\left(\tfrac{1}{2}(4)(2)\right)$$
$$= -4$$