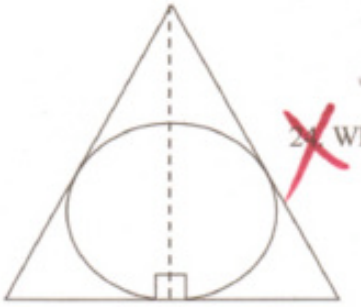
Harry Potter

and the

Calculus House Cup



2. What is the dotted line called? the elder wand

May your Geometry students never give this answer.

Presenter

Denise Fuji McCleary

http://www.zendog.org/homework

How to do a House Cup Competition

How the points are awarded

After each quiz or test calculate the percentage of A's. Each percentage point is worth one point.

The House with the most points wins a party after the AP exam given by me. [And bragging rights] The other houses may have a party but must bring their own refreshments.

1. The table below gives values of the velocity, $v(t)$, of a Inferi [An "inferi" is sort of like a zombie but made by the Dark Lord] at selected times.

t *(sec)*	0	1	3	6	10	15
$v(t)$ *m/sec*	5	1	-1	5	10	13

(a) Is there a time during $0 < t < 15$, that the velocity is equal to 9 *m/sec*

Justify completely.

(b) Find an approximation for the acceleration at time $t = 2$ and indicate units. [Show all work]

(c) Show that there must be a time interval such that the acceleration, $a(t)$, is equal to zero

(d) **If *s(1)= 10 meters,* then write the equation of the line tangent to the graph *of s(t) at t = 1.* Then use the tangent line to find an approximation for the value of *s(1.1)***

(e) **Use a Left Riemann Sum using the intervals indicated by the data, to find an approximation for $\int_0^{15} v(t)\,dt$ [Indicate units and show all work]**

All pictures are from Google Images

Dobby and Kreacher

Two house elves move along a straight path. For $0 \le t \le 6$, the position of Dobby is given by the function $d(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of Kreacher is given by $k(t) = t^3 - 6t^2 + 9t + 3$

(a) For $0 \le t \le 6$, find all times during which Kreacher is moving to the right.

(b) Find Dobby's acceleration at time *t = 3*. Is Dobby speeding up, slowing down, or doing neither at this time?

Professor Umbridge has asked Argus Filch to once again hang a new decree from the Ministry of Magic. The ladder is 13 meters long and is leaning against the wall. Those silly Weasley twins are hiding around a corner and using magic to slowly move the bottom of the ladder from the bottom of the wall at a rate of 1 meter per second. At what rate is the top of the ladder moving down the wall when it is 12 meters above the ground?

When Harry Potter was young, he got upset and "accidentally" turned his Aunt Marge into a balloon. The *rate of change*, *R*, in kilometers per hour, of the altitude of the hot air balloon is given by $R(t) = t^3 - 4t^2 + 6$ for time $0 \leq t \leq 4$, where *t*, is measured in hours. Assume the balloon is initially at ground level.

(a) At what time(s) is Aunt Marge neither ascending nor descending?

(b) At what time during the interval $0 \leq t \leq 4$ does Aunt Marge reach her relative maximum height?

The Room of Requirement

During the many centuries of its use, the Room of Requirement [or the Room of Hidden Things] in Hogwarts Castle has accumulated tons of objects that students and teachers have hidden. At the beginning of 1991, there were 35 tons of objects. [Hey, the room is magical!] The increasing function M models the total amount of objects in the magical room. The headmaster estimates that M will satisfy the differential equation $\frac{dM}{dt} = \frac{1}{7}(M - 14)$ for the next 7 years. M is measured in tons, and t is measured in years from 1991.

(a) Use the line tangent to the graph of M at $t = 0$ to approximate the amount of objects that the room contains at the end of the first 4 months of 1991. [time $t = \frac{1}{3}$] ***[non-calculator]***

(b) **Find $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of objects that the room contains at time $t=\frac{1}{3}$.**

(c) **Find the particular solution $M=M(t)$ to the differential equation $\frac{dM}{dt}=\frac{1}{7}(M-14)$ with initial condition $M(0)=35$.**

Potter Potion Problems to Ponder

A 1200 ml cauldron [big pot] of Polyjuice Potion is filled to capacity. [You can find the recipe at: http://harrypotter.wikia.com/wiki/Polyjuice_Potion]

At time t = 0, the potion magically begins to drain out of the cauldron at a rate modeled by $r(t)$,measured in milliliters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{50t}{t+5} & for\ 0 \le t \le 5 \\ 70e^{-0.2t} & for\ t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which the Polyjuice potion is draining from the cauldron from the tank between time $t = 0$ and time $t = 10$

(c) Find $r'(1)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Find the amount of potion in the tank at time t = 3 hours.

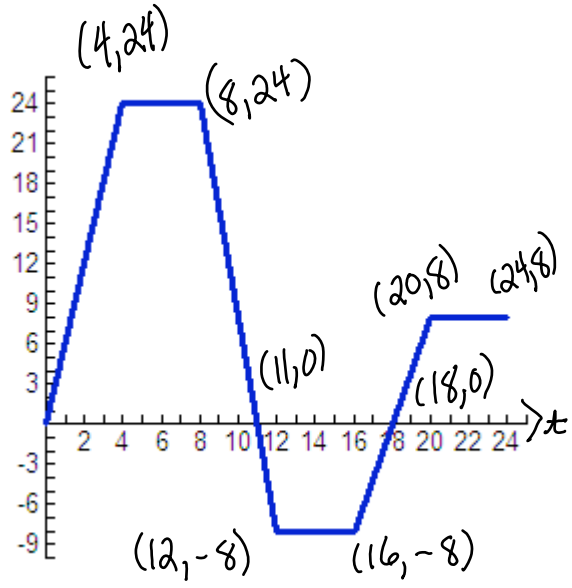
(e) Find the amount of potion in the tank at time t = 24 hours.

More Potter Problems to Ponder [#1 = non-calculator, #2 = calculator]

Study Buddies _____

Graph of $v(t)$

While sneaking through the Forbidden Forest at Hogwarts, Harry and Luna observe a unicorn traveling on a straight path. The path leads from a Skittles tree to a stream. [Unicorns eat Skittles.] For $0 \le t \le 24$ the unicorn's velocity is modeled by the piecewise-linear function defined by the graph below.



Graph of $v(t)$

(a) At what times in the interval $0 \le t \le 24$, if any, does the unicorn change direction? Give a Calculus-based reason for your answer.

(b) Find the total distance the unicorn travels during the time interval $0 \le t \le 24$.

(c) At what time in the interval $0 \le t \le 24$ is the unicorn the farthest from the Skittles tree? How far from the Skittle tree is the unicorn at that time? [You may assume that the unicorn started at the Skittles tree]

(d) Write expressions for the unicorn's acceleration $a(t)$, velocity $v(t)$, and the position with respect to the Skittles tree that are valid for the time interval $8 < t < 12$

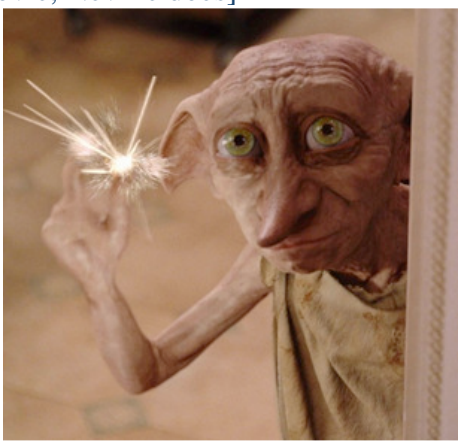
(2) Gillyweed grows [in pounds] in the lake at Hogwarts. The rate of growth of the gillyweed is proportional to the amount of gillyweed. [Note: Gillyweed was the seaweed-like herb that Harry Potter ate in order to be able to breathe underwater during the Tri-Wizard Tournament.]

(a) Find an expression for G, the amount of gillyweed in the lake [in pounds], in terms of t, the number of years, if the amount of gillyweed is 100 pounds initially and 120 pounds after one year.

(b) In how many years will the amount of gillyweed available be 300 pounds?

(c) Find the derivative to the function you found in part (a) and use it to write the equation of the tangent line to G at the point (0, 100). Then use the tangent line equation to predict the value at $t = 1$. Was this an over- or an under-estimate?

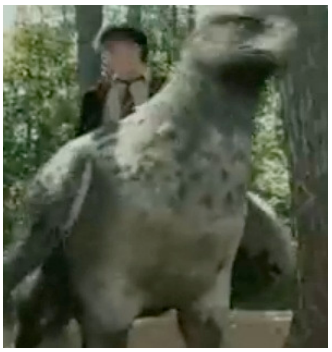
[In the book, Dobby gives Harry the gillyweed – in the movie, Neville does]



Harry Potter rides the hippogriff, Buckbeak, back and forth along a straight path in the Forbidden Forest. The twice-differentiable function *H* models Harry's *position* on the path, measured in meters from the beginning of the path, at time *t*, measured in seconds from the start of the ride. The table below gives values for *H(t)* and Harry's velocity, *v(t),* measured in meters per second at selected time.

t *(seconds)*	0	15	30	60	100	120
H(t) *(meters)*	50	100	25	10	50	75
v(t) *(meters/sec)*	3	3.2	4	4.5	-5	-10

From Google Images



(a) Use the data in the table to find an estimate for Harry's acceleration at time t = 10 seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{120} |v(t)|\,dt$. Approximate $\int_0^{120} |v(t)|\,dt$ using the trapezoid method with the subintervals indicated by the data in the table.

(c) Must there be a time, $0 \le t \le 120$, when Harry's velocity is equal to zero? Explain why or why not.

(d) What is the average acceleration during the time interval $0 \le t \le 120$?

Also from Google Images

Harry Potter and the Deathly Derivatives

Inspired by: http://www.youtube.com/watch?v=bpZ6fSuP9dw

Lord Voldemort is dead and the Death Eaters have disbanded. But all is still not well in the magical world. A small group of former Death Eaters have formed an evil band called the Deathly Derivatives who torture Muggles and wizards with Calculus. First they immobilize you [with the Incarcerous spell] and then force you to find derivatives using Calculus. Too bad they don't teach AP Calculus at Hogwarts! [Or any mathematics for that matter] Find the derivatives of the following and use their solutions to unlock the spell that binds you.

Find the derivatives of the functions given on the back of this page. The answer to each problem will match a letter that will allow you to find the spell to overthrow the "Deathly Derivatives"

[From Google Images]

Do all work on a separate piece of paper. Show ALL steps.

1.	$\sqrt{x^2+7}$	**Y.**	$\cos(3x)[\sin(3x)]^{-2/3}$
2.	$\sin^2(3x)$	**D.**	$6\sin(3x)\cos(3x)$
3.	$\dfrac{\cos(3x)}{\sin(3x)}$	**S.**	0
4.	$\sec^2(3x)$	**A.**	$3\sec^2(3x)$
5.	$\dfrac{1}{\sqrt{x^2+7}}$	**O.**	$-x\left(x^2+7\right)^{-3/2}$
6.	$\tan(3x)$	**X.**	$3\cos^2(3x)-3\sin^2(3x)$
7.	$\sin(3x)$	**M.**	$-3\csc^2(3x)$
8.	$\sin(3x)\cos(3x)$	**I.**	$\dfrac{x}{\sqrt{x^2+7}}$
9.	$\sqrt[3]{\sin(3x)}$	**U.**	$3\cos(3x)$
10.	$\sin^2(3x)+\cos^2(3x)$	**T.**	$6\sec^2(3x)\tan(3x)$

___	___	___	___	___	___	___	___
10	**4**	**7**	**2**	**1**	**5**	**7**	**10**

___	___	___	___	___	___	___
3	**6**	**8**	**1**	**3**	**7**	**10**

A Potpourri of Potter Problems

[Questions 1 and 2 were found on Youtube from "mrpenguinbobman"]

1. Harry and Malfoy are having a grudge match to decide who is the better wizard. Harry uses an EXPELLIARMUS charm and knocks Malfoy's wand straight up into the air at an initial velocity of 16 ft/sec. What would the maximum height the wand would reach if its initial position is 4 feet and acceleration due to gravity is $-32\,ft/\sec^2$? [Solve using Calculus]

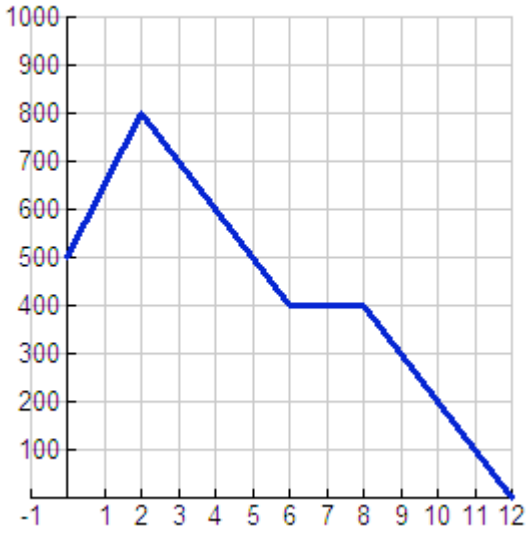
2. The rate at which wizards pass through the wall into Platform $9\frac{3}{4}$, measured in wizards per minute, is modeled by the function $F(t) = 24 + 2\sin\left(\frac{t}{2}\right)$ for the times the platform is open, $0 \le t \le 30$ in minutes.

To the nearest whole number, how many wizards enter the platform in the 30 minute period?

3. It seems like it rains a lot at Hogwarts. The rainfall rate in cm per hour was measured during a 10-hour storm. Use the trapezoid rule to estimate the total amount of rain during the storm.

Time [hours]	0	1	3	4	5	6	7	8	10
Rate in cm/hour	0	0.5	1	0.2	1.5	2	0.3	1	1

Harry Potter at the Movies



Time (in hours)

At midnight, there are 1000 people in line to buy tickets for the opening night of the latest Harry Potter movie at the local multi-screen movie theatre when the box office begins selling tickets for the first showing. Once the movie begins, the theatre continues to sell tickets for later screenings until noon when an unexpected power outage takes place. While there is a line, people enter the theatre at a rate of 500 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the theatre throughout the day. Time t is measured in hours from the time the theatre starts selling tickets.

(a) How many people arrive to buy tickets between $t = 0$ and $t = 5$?

Show the computations that lead to your solution.

(b) Is the number of people waiting in line to buy tickets increasing or decreasing between $t = 2$ and $t = 4$? Justify your answer.

(c) At what time t is the line for tickets the longest? How many people are in line at that time? Justify your answer.

Double Trouble [a pair activity]

2011doubletrouble.doc

All of the following problems require you to use at least two different differentiation rules. Like the Weasley twins, they might be trouble! Show ALL steps and use standard mathematical notation.

Find the derivative of the following functions:

1. $y = \sqrt{x^2 + 1}\,\sin(2x)$

2. $f(x) = \sin^2(\pi x)$

3. $g(x) = \cos^4\left(\sqrt{\pi x}\right)$

4. $h(x) = \dfrac{\sqrt{x^4 + 4}}{\cot x}$

5. $y = \dfrac{x^3 + 1}{\sec(2\pi x)}$

6. $h(x) = \dfrac{f(20x)}{g(11x)}$

7. $f(t) = g(5t)\,h(2t) - h(5t)\,g(2t) + g^{2}(t)\,h^{3}(t)$

8. **Find** $\frac{dy}{dx}$

$\cos(x\,y) = x\,y$

Harry Potter and the Related Rates

While under his cloak of invisibility, Harry Potter throws a snowball at Draco Malfoy. If the snowball is melting at the rate of 10 cubic centimeters per minute, at what rate is the radius changing when the snowball's radius is 12 centimeters?

Cho Chang is traveling north towards an intersection on her Comet 260 [flying broomstick] at a rate of 60 kilometers per hour, while Harry Potter is traveling east away from the intersection on his Nimbus 2000 [flying broomstick] at a rate of 50 kilometers per hour. Find the rate of change between Cho and Harry when Cho is 3 kilometers south of the intersection and Harry is 4 kilometers east of the intersection.

In Potions Class, a potion runs into a conical container at the rate of $9\ cm^3 / \min$. The cone stands down and has a height of 10 cm and a base radius of 5 cm. How fast if the potion level rising when the potion is 6 cm deep?

Harry has accidentally turned is Aunt Marge into a balloon. Aunt Marge is rising vertically above a level, straight road at a constant rate of 1 m/sec. Just when the balloon is 65 meters above the ground, a bicycle moving at a constant rate of 17 m/sec passes under Aunt Marge. How fast is the distance between Aunt Marge and the bicycle increasing 3 seconds later.