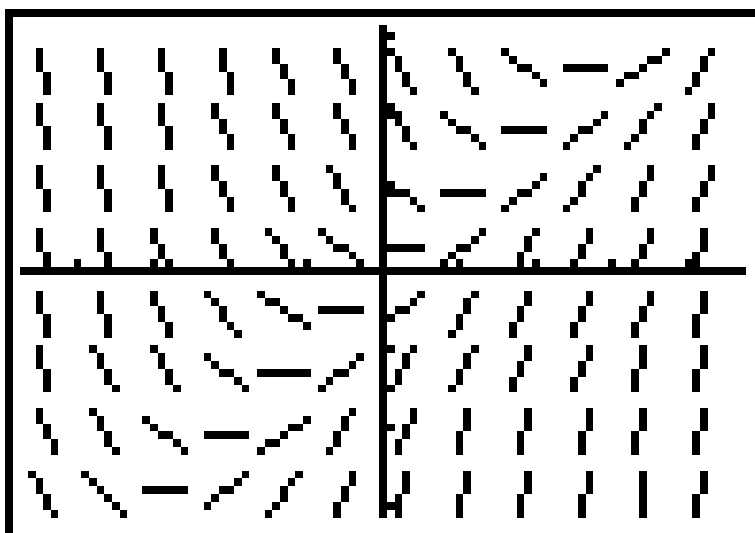


Slope Field Problems and Examples

as compiled by Lin McMillan



Problems and Examples

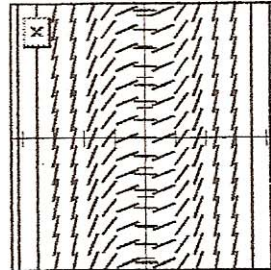
1. 1998 AP Exam BC 24, Free Response 4 (draw slope field + Euler + full solution)
2. 2000 AP Exam BC 6 (draw slope field + interpret slope field + analytic solution + use solution and slope field together) – Good question. (See page 9)
3. 2002 AP Exam BC (See page 10)

The questions below are taken Multiple-Choice and Free Response Questions in Preparation for the AP Calculus (BC) Examinations, by David Lederman assisted by Lin McMullin, D&S Marketing Systems, Inc. 1205 38th Street, Brooklyn, NY 11218. © 1999 (Phone 1-800-633-8383; On-line at www.dsmarketing.com). You may use them in your classroom.

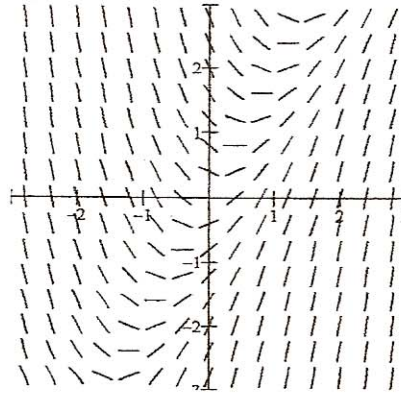
The slope fields were drawn with Winplot available *free* at <http://math.exeter.edu/rparris>. This program also does graphs (rectangular, polar, parametric, implicit plots) in 2D and 3D. It also draws solids of revolution and solids with regular cross sections. Did I mention it's FREE?

4. The slope field for the differential equation $dy/dx = f(x)$ is shown below for $-4 < x < 4$ and $-4 < y < 4$. Which statement is true for all possible solutions of the differential equation?

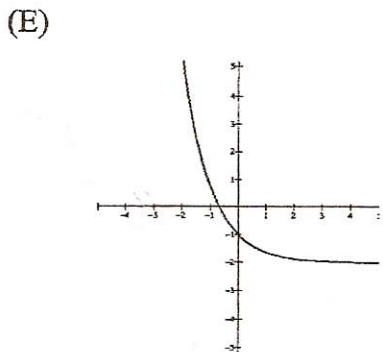
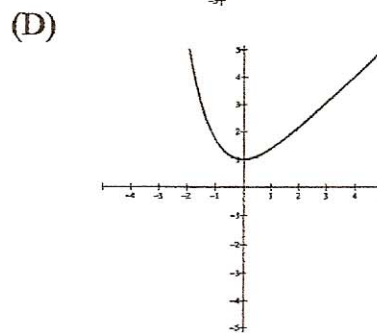
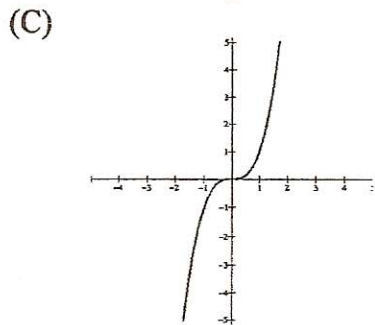
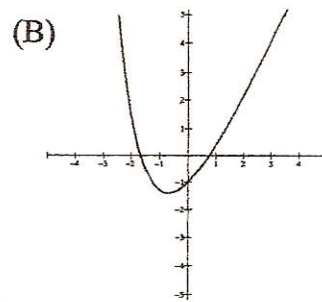
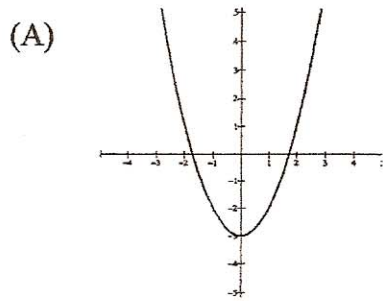
- I. For $x < 0$ all solutions are decreasing.
 - II. All solutions level off near the y -axis
 - III. For $x > 0$ all solutions are increasing.
- (A) I only (B) II only (C) III only (D) II and III only (E) I, II and III



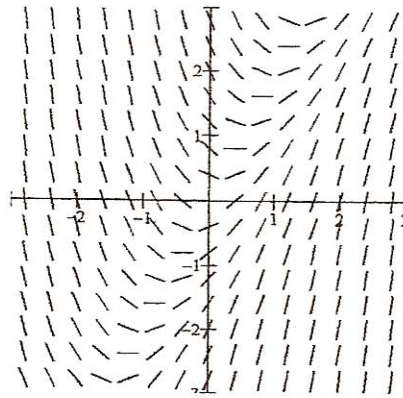
5. Slope field $y' = 2x - y$



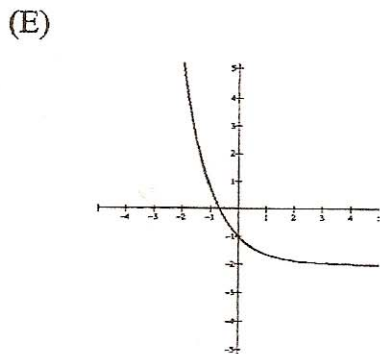
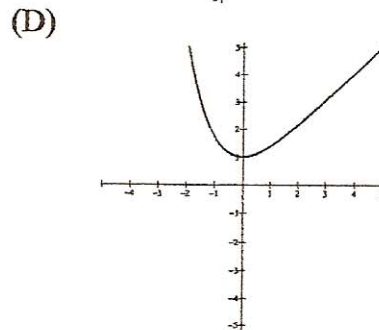
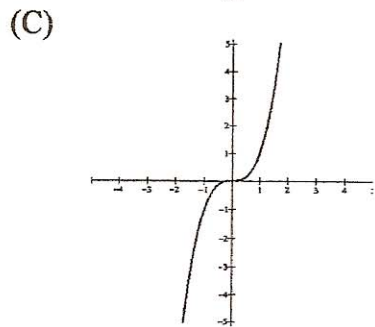
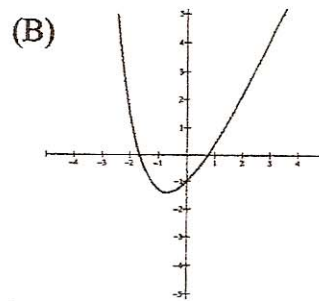
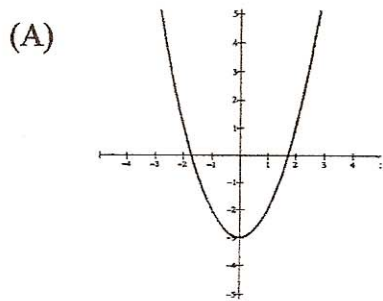
Which graph could be a solution of the differential equation that is shown above?



5. Slope field $y' = 2x - y$



Which graph could be a solution of the differential equation that is shown above?



6. The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2y + y^2x}{3x + y}$ will have horizontal segments when

segments when

(A) $x = 0$ or $y = 0$, only

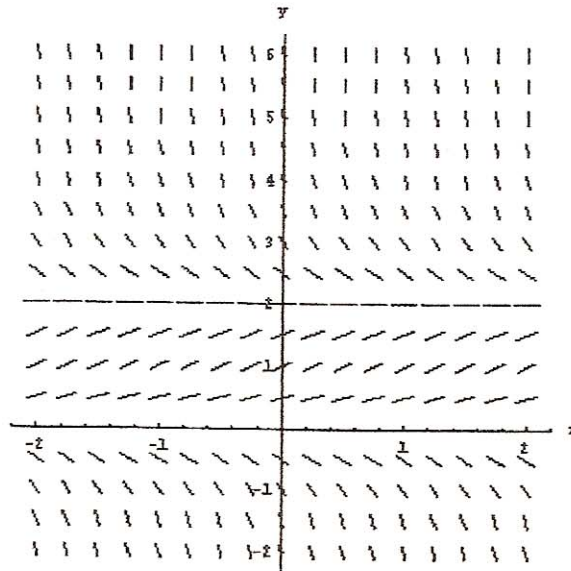
(B) $y = -x$, only

(C) $y = -3x$, only

(D) $y = 5$, only

(E) $x = 0$, or $y = 0$, or $y = -x$

7.



The slope field for a differential equation $\frac{dy}{dx} = f(y)$ is shown in the figure above.

Which statement is true about $y(x)$?

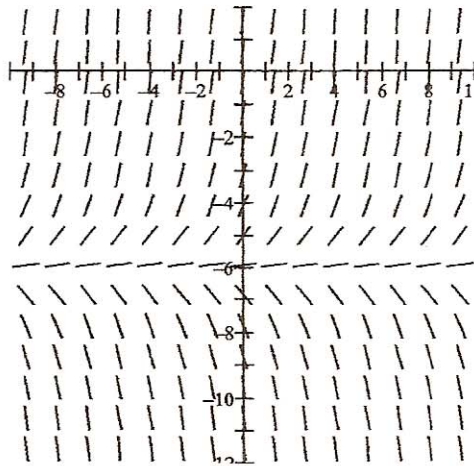
I. If $y(0) > 2$ then $\lim_{x \rightarrow \infty} y(x) \approx 2$

II. If $0 < y(0) < 2$ then $\lim_{x \rightarrow \infty} y(x) \approx 2$

III. If $y(0) < 2$ then $\lim_{x \rightarrow \infty} y(x) \approx 2$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II and III

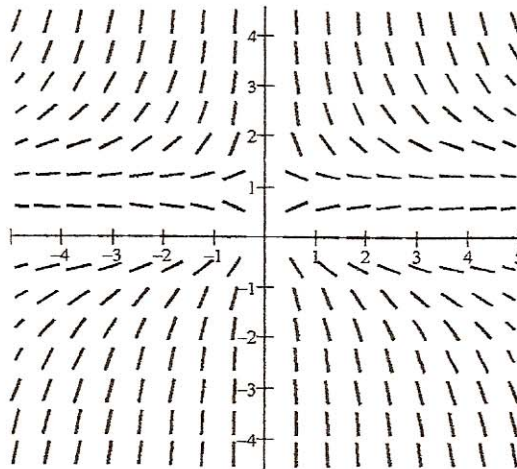
8. Consider the differential equation $dy/dx = 6 + y$ defined for all real numbers. The slope field for this equation is shown below in the window $-10 < x < 10$ by $-12 < y < 2$.



- (a) Find the general solution of the differential equation in terms of an arbitrary constant C .
- (b) Find the particular solution of the differential equation that meets the initial condition that $y = 0$ when $x = 0$. Sketch this solution on the slope field.
- (c) Find the particular solution of the differential equation that meets the initial condition that $y = -8$ when $x = 0$. Sketch this solution on the slope field.
- (d) The slope field indicates that for some of the solutions $\lim_{x \rightarrow \infty} y = \infty$ and for other solutions $\lim_{x \rightarrow \infty} y = -\infty$. Determine the values of C for which $\lim_{x \rightarrow \infty} y = \infty$. Show your reasoning.

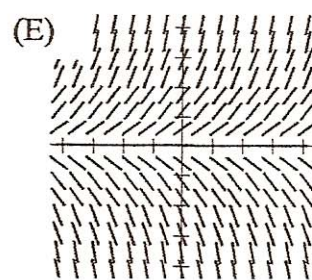
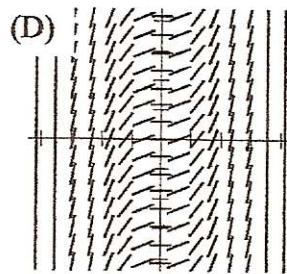
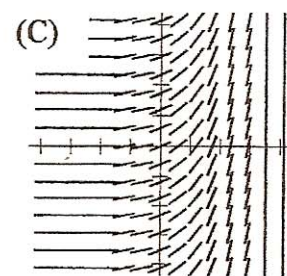
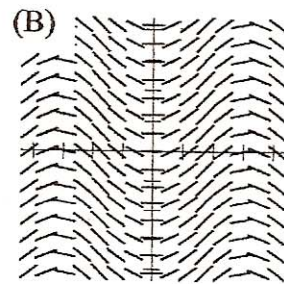
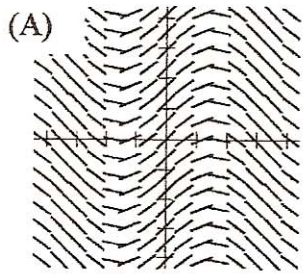
9. Consider the differential equation $\frac{dy}{dx} = \frac{y-y^2}{x}$ for all $x \neq 0$.

- (a) Verify that $y = \frac{x}{x+C}$, $x \neq -C$ is a general solution for the given differential equation and show that all solutions contain $(0,0)$.
- (b) Write an equation of the particular solution that contains the point $(1, 2)$. and find the value of $\frac{dy}{dx}$ at $(0,0)$ for this solution.
- (c) Write an equation of the vertical and horizontal asymptotes of the particular solution found in (b).
- (d) The slope field for the given differential equation is provided. Sketch both branches of the particular solution curve that passes through the point $(1,2)$.



10. Which choice represents the slope field for $dy/dx = \sin x$?

Note: All graphs are for $-4 < x < 4$ and $-4 < y < 4$



Answers:

4. D 5. B 6. E 7. D

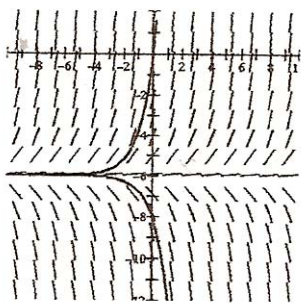
8 (a) $y = Ce^x - 6$ (b) $y = 6e^x - 6$ (c) $y = -2e^x - 6$ (d)

$C > 0$ 9. (b) The particular solution is $y = \frac{x}{x - \frac{1}{2}}$, and

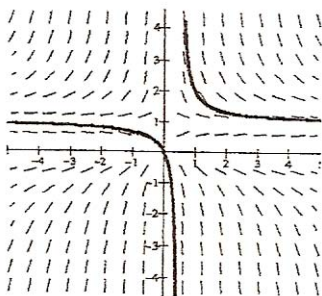
the slope at $(0,0)$ is $y'(0) = \frac{-\frac{1}{2}}{(0 - \frac{1}{2})^2} = -2$ The

asymptotes are $x = \frac{1}{2}$ and $y = 1$ 10. B

8 b and c



9 d.



10. B