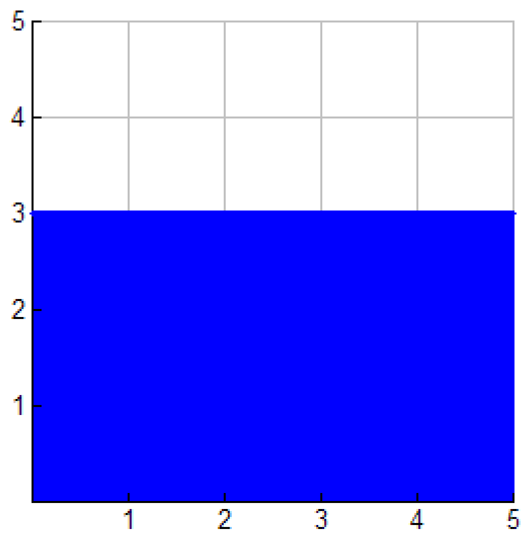
Definite Integrals

If $f(x) > 0$ on $[a,b]$, then we can think of $\int_a^b f(x)\,dx$ to be the area between the curve and the $x-$axis.

So far we have used rectangles to find the area [LRAM, RRAM, MRAM, Riemann Sums]. We can use other geometric formulas if we have "nice" curves.

Let's consider page 278 #13-18 [set up definite integrals to represent the shaded region]

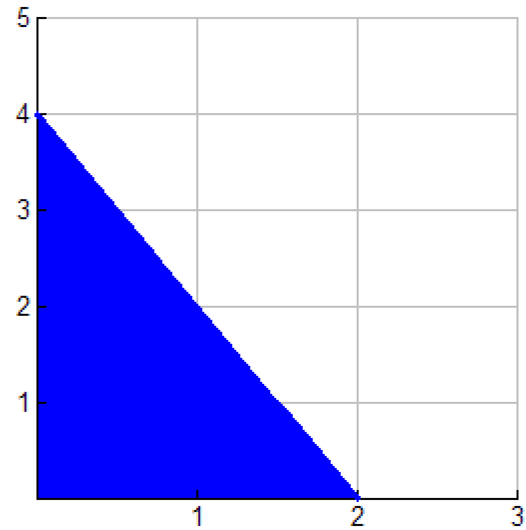


$y = 3$

$$\int_0^5 3\,dx$$

$= 3(5)$

$= 15$

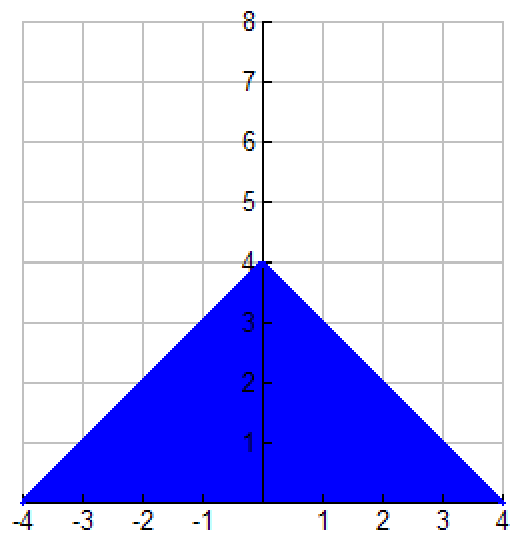


$$\int_0^2 (4-2x)\,dx$$

$$= \tfrac{1}{2}(2)(4)$$

$$= 4$$

$y = 4 - 2x$



$$\int_{-4}^{4} (4-|x|)\,dx$$

$$= \tfrac{1}{2}(8)(4)$$

$$= 16$$

$y = 4 - |x|$

16. $f(x) = x^2$

$$\int_0^2 x^2\,dx$$

17. $f(x) = 4 - x^2$

$$\int_{-2}^2 (4-x^2)\,dx$$

18. $f(x) = \dfrac{1}{x^2+1}$

$$\int_{-1}^1 \frac{1}{x^2+1}\,dx$$

Rules of Definite Integrals [$f(x)$ must be integrable]

$a \in \mathbb{R}$

$$\int_a^a f(x)\,dx = 0$$

ex $\int_1^1 x\,dx = 0$

On $[a,b]$ which means that $a < b$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

LOWER BOUND MUST BE ON THE "BOTTOM"

ex $\int_1^0 x\,dx$

$= -\int_0^1 x\,dx$

If $a < c < b$, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Ex $\int_5^{12} f(x)dx = \int_5^{7} f(x)dx + \int_7^{12} f(x)dx$

If $k \in$ reals, then $\int_a^b k\ f(x)\,dx = k \int_a^b f(x)\,dx$

[Constant multiple rule]

Ex $\int_\pi^5 2f(x)dx = 2\int_\pi^5 f(x)\,dx$

Sum or Difference Rule

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

Let's try out our new rules:

think $\int_2^4 1\,dx$

$\int_2^4 x^3\ dx = 60$ $\quad\quad$ $\int_2^4 x\ dx = 6$ $\quad\quad$ $\int_2^4 dx = 2$

1. $\int_4^4 x^3 dx = 0$

2. $\int_4^2 dx = -\int_2^4 dx$

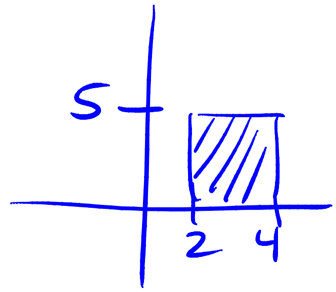
$= -2$

3. $\int_2^4 5\,dx$

$= 5\int_2^4 dx$

$= 5(2)$

$= 10$



4. $\int_2^4 [x+2]\,dx$

$= \int_2^4 x\,dx + 2\int_2^4 dx$

$= 6 + 2(2)$

$= 10$

5. $\int_2^4 \left[\frac{1}{2}x^3 - 2x + 4\right] dx$

$= \frac{1}{2}\int_2^4 x^3\,dx - 2\int_2^4 x\,dx + 4\int_2^4 dx$

$= \frac{1}{2}(60) - 2(6) + 4(2)$

$= 26$

Now let's try page 279 # 42

Given: $\int_0^3 f(x)\,dx = 4$ AND $\int_3^6 f(x)\,dx = -1$

$\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx$

$= 4 + -1$

$= 3$

$$\int_6^3 f(x)\,dx$$

$$= -\int_3^6 f(x)\,dx$$

$$= -(-1)$$

$$= 1$$

$$\int_3^3 f(x)\,dx = 0$$

$$\int_3^6 -5\,f(x)\,dx$$

$$= -5\int_3^6 f(x)\,dx$$

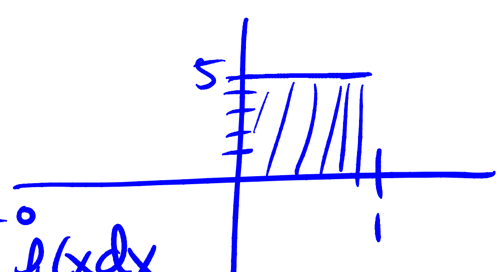
$$= -5(-1) = 5$$

If time, see #44

$$\int_{-1}^1 f(x)\,dx = 0 \qquad \int_0^1 f(x)\,dx = 5$$

EASY SCENARIO

(a) $$\int_{-1}^0 f(x)\,dx$$



$$\int_{-1}^1 f(x)\,dx - \int_0^1 f(x)\,dx = \int_{-1}^0 f(x)\,dx$$

$$0-5 = -5$$

(b) $$\int_0^1 f(x)\,dx - \int_{-1}^0 f(x)\,dx$$

$$= 5 - (-5)$$

$$= 10$$

(c) $$\int_{-1}^1 3\,f(x)\,dx$$

$$= 3\int_{-1}^1 f(x)\,dx$$

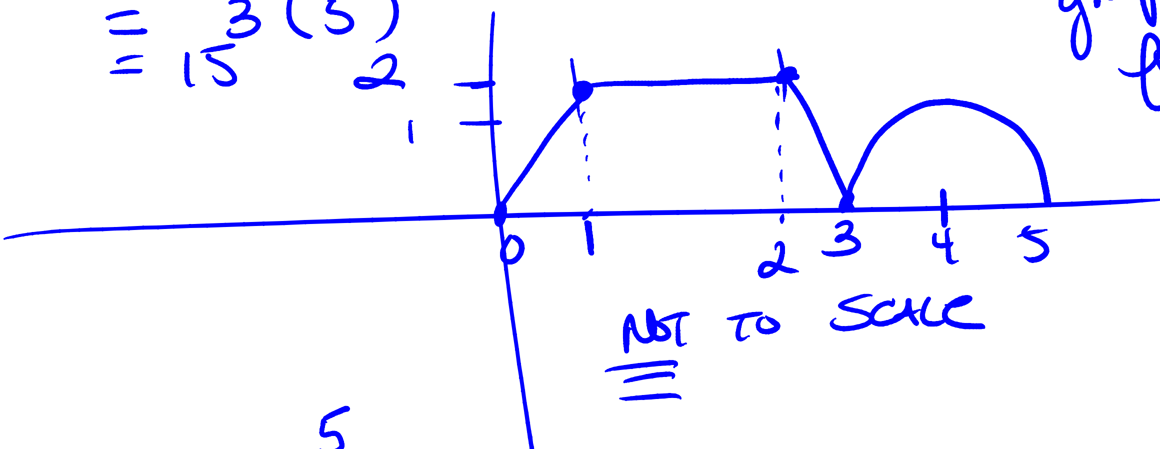
$$= 0$$

(d) $$\int_0^1 3\,f(x)\,dx$$

$$= 3\int_0^1 f(x)\,dx$$

$$= 3(5)$$

$$= 15$$



graph of $f(x)$

find $$\int_0^5 f(x)\,dx$$

$$= \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx$$

$$= \frac{3+1}{2}(2) + \frac{\pi}{2}(1^2)$$

$$= 4 + \frac{\pi}{2}$$

Homework: page 279 # 23, 27, 33, 35, 37, 41, 43

use our new rules

please show great notation