How to find a limit if direct substitution fails [and you do not have a calculator]

If we use direct substitution and obtain the indeterminate form $\frac{0}{0}$, then it may be possible to do something *algebraic*. See page 62, Theorem 1.7.

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

If we let $x = 2$, then we get an indeterminate form. Let's consider using Theorem 1.7.

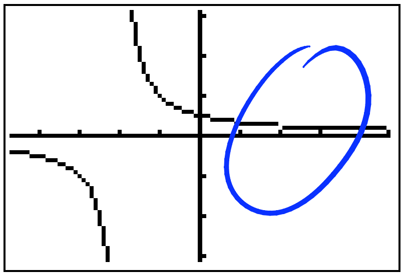
Please notice the standard notation

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

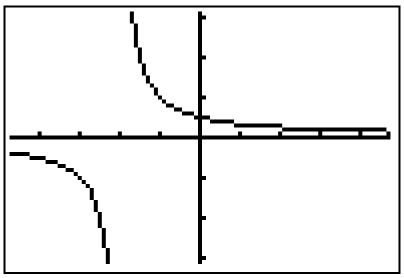
$$= \lim_{x \to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)}$$

$$= \lim_{x \to 2} \frac{1}{x+2}$$

$$= \frac{1}{4}$$



graph of $y = \frac{x-2}{x^2-4}$



graph of $y = \frac{1}{x+2}$

Find: $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2}$

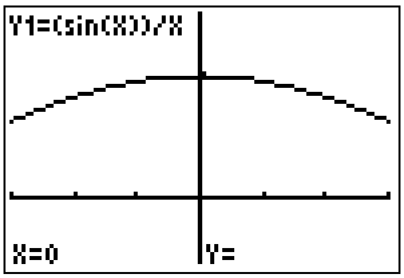
$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{x-2}}$

$= \lim_{x \to 2} (x-1)$

$= 1$

Consider:

$\lim_{x\to 0}\frac{\sin x}{x}$ If we use direct substitution, then we get the indeterminate form $\frac{0}{0}$, but nothing algebraic can be done. Let's use our TI to see if a limit exists.



The graph looks continuous at x = 0 but it is not!
Use a table to find the limit.

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Find:

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

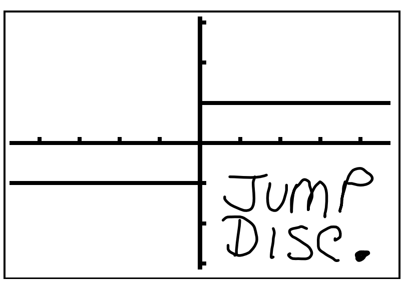
WE used OUR TRUSTY TI

♪ There is another analytic method that we can use to find these limits but we need to learn the next chapter in order to do it.

Consider:

$$\lim_{x\to 0}\frac{|x|}{x}$$ Does direct substitution work?

How about a graph?



$$\lim_{x\to 0}\frac{|x|}{x} = \text{d.n.e.}$$

you need to know this graph

Which of the following can be done with direct substitution?

(1) $\lim_{x\to 0}\tan x = 0$

(2) $\lim_{x\to 0}\frac{\sin(2x)}{2x} = 1$ TABLE OR GRAPH

(3) $\lim_{x\to 0}\frac{x-1}{x^2-1} = 1$

(4) $\lim_{x\to 5} \frac{x^2-25}{x-5}$

$= \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}}$

$= \lim_{x\to 5} (x+5)$

$= 10$

(5) $\lim_{x\to 2} \frac{|x|}{x} = 1$

(6) $\lim_{x\to 1} \frac{x^2+2x+3}{x^2+5} = \frac{6}{6}$

(7) $\lim_{x\to 0} \frac{x}{\cos x} = 0$

(8) $\lim_{x\to 0} \frac{1}{x}$ d.n.e.

INFINITE DISC.

(9) $\lim_{x\to 1} \frac{x^3-1}{x-1}$

$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$

$= \lim_{x\to 1} (x^2+x+1)$

$= 3$

WE HAVE A LIMIT
BUT NOT A FUNCTION
VALUE $\longrightarrow$ REMOV. DISC.
Point $(1,3)$

Consider:

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

[Sometimes written with an h instead of a Δx]Does direct substitution work? NO If not, can we do something algebraic?

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[\cancel{x^2} + 2x\Delta x + (\Delta x)^2] - \cancel{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x)$$

$$= 2x$$

♪ The Squeeze Theorem is not part of the AP curriculum

Homework on page 68:

Do analytically: # 45, 46, 47, 48, 49, 59

Do graphically [show graph] # 67, 71

Please be mindful of your notation