Implicit Differentiation and Related Rates Review

Implicitreview.doc

Common implicit differentiation examples:

$$\frac{d}{dx}\ y = \frac{dy}{dx}$$

$$\frac{d}{dx} y^2 = 2\,y\frac{dy}{dx}$$

$$\frac{d}{dx} xy = y + x\frac{dy}{dx}$$ [Got to love the Product Rule]

$$\frac{d}{dx}\left(\frac{x}{y}\right) = \frac{y - \ x\dfrac{dy}{dx}}{y^2}$$ [Got to love the Quotient Rule]

$$\frac{d}{dx}\sin(xy) = [\cos(xy)]\left[y + x\frac{dy}{dx}\right]$$

[Got to love the Chain Rule]

$$\frac{d}{dx} e^y = e^y\,\frac{dy}{dx}$$

$$\frac{d}{dx} \ln y = \frac{1}{y}\ \frac{dy}{dx}$$

Common implicit differentiation questions:

If $x + y = xy$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x-1}$

(B) $\frac{y-1}{x-1}$

(C) $\frac{1-y}{x-1}$

(D) $x - y - 1$

(E) $\frac{2-xy}{y}$

If $e^{xy} = 2$, then at the point $(1, \ln 2)$, $\frac{dy}{dx} =$

(A) $-\ln 2$

(B) $2\ln 2$

(C) $\ln 2$

(D) $-2e$

(E) $-4\ln 2$

Related Rates

Finding the rate at which a quantity is changing based on other known rates. These are the problems where we took $\frac{d}{dt}$ of all of the terms of some equation.

Find $\frac{d}{dt}$ of $A = \pi r^2$

The volume of a cube is increasing at the rate of 20 cubic centimeters per second. How fast, in square centimeters per second, is the surface area of the cube increasing at the instant when each edge of the cube is 10 centimeters long?

(A) $\frac{4}{3}$
(B) 2
(C) 4
(D) 6
(E) 8

If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?

(A) 40π
(B) 80π
(C) 800
(D) 800π
(E) 3200π

The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at (2, 1) is

(A) $\frac{3}{2}$
(B) $\frac{-5}{14}$
(C) $\frac{-3}{14}$
(D) 0
(E) -1

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5. Consider the curve given by $y^2 = 2 + xy$.

   (a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

   (b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

   (c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

   (d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

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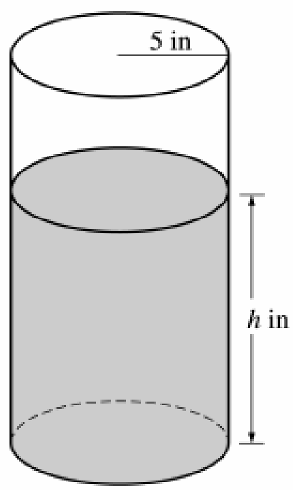

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?