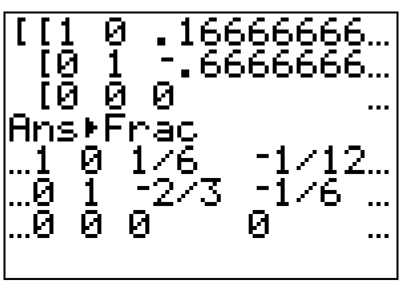
I think that the following is a better solution than the one I did in class.

Find the vector equation of the line of intersection of the three planes represented by the following system of equations

This is a G.D.C. problem

$$\begin{aligned} 2x \quad &- 7y \quad + 5z = 1 \\ 6x \quad &+ 3y \quad - z = -1 \\ -14x \quad &- 23y \quad + 13z = 5 \end{aligned}$$

With m y trusty TI I found



This confirms that there are an infinite number of solutions!

Now I will take the cross product of the first two planes [Note: It does not make a difference which two planes you take. I just picked the ones with the smaller coefficients]

$$\begin{vmatrix} i & j & k \\ 2 & -7 & 5 \\ 6 & 3 & -1 \end{vmatrix}$$

The cross product is: $-8i + 32j + 48k$

If we divide the coefficients by 48, we get:

$$-\frac{1}{6}i + \frac{2}{3}j + k$$

[Note: Some people like fractions!]

Now to hunt for a common point. Let z = 0

2x-7y=1

6x+3y= - 1 Solve to get: $y = -\frac{1}{6},\ x = -\frac{1}{12}$

So our point is: $\left(-\frac{1}{12}, -\frac{1}{6}, 0\right)$

Now we have a point and a direction vector. Hence, thus, therefore, our line of intersection is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{12} \\ -\frac{1}{6} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{1}{6} \\ \frac{2}{3} \\ 1 \end{pmatrix}$$

They had this as an intermediate step. I think it should be the last step. Go USA!