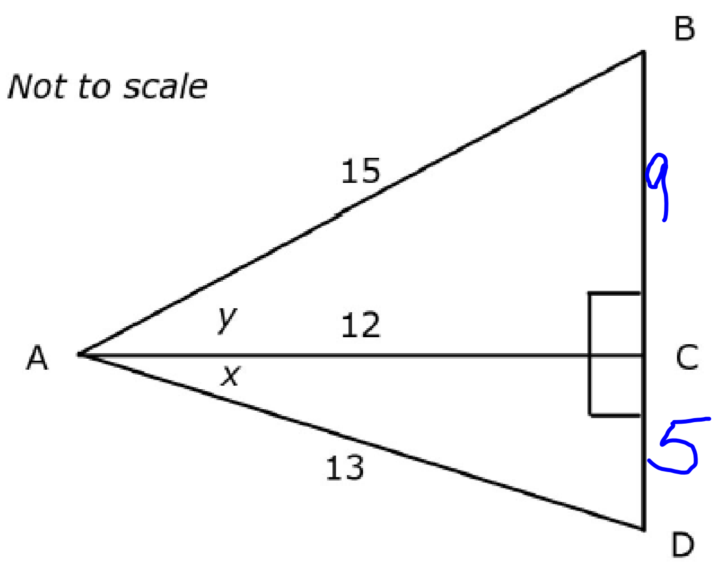
Our trigonometry revision of slightly strange problems.

Our textbook has several chapter devoted to trig. Also, the formula packet has a lot of trig stuff.

Let's warm up on some easier problems, then work our way through some more difficult ones.



$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$$

Use the diagram above to find the exact value of $\tan(x - y)$.

$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$$

$$= \frac{\frac{5}{12} - \frac{9}{12}}{1+\left(\frac{5}{12}\right)\left(\frac{9}{12}\right)}$$

$$= \frac{-\frac{4}{12}}{1+\frac{45}{144}}$$

$$= \frac{-\frac{1}{3}}{\frac{189}{144}}$$

$$= -\frac{16}{63}$$

Using Sine Models [from our textbook] Page 280, 281

Example 10

The height $h(t)$ metres of the tide above mean sea level on January 24th at Cape Town is modelled approximately by $h(t) = 3\sin\left(\frac{\pi t}{6}\right)$ where t is the number of hours after midnight.

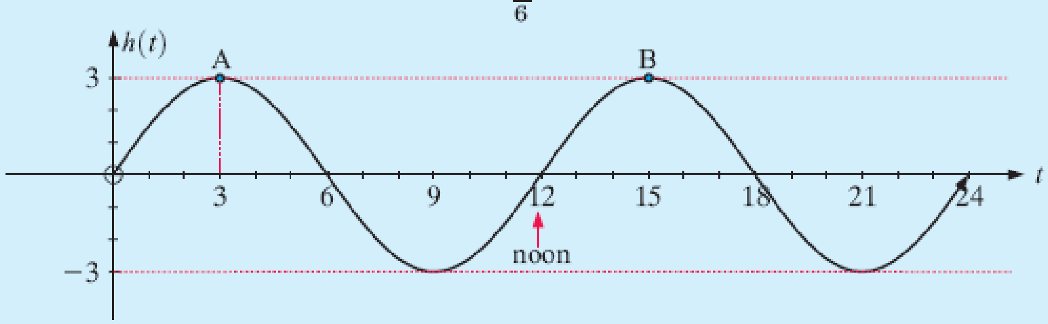
a Graph $y = h(t)$ for $0 \leqslant t \leqslant 24$.

b When is high tide and what is the maximum height?

c What is the height at 2 pm?

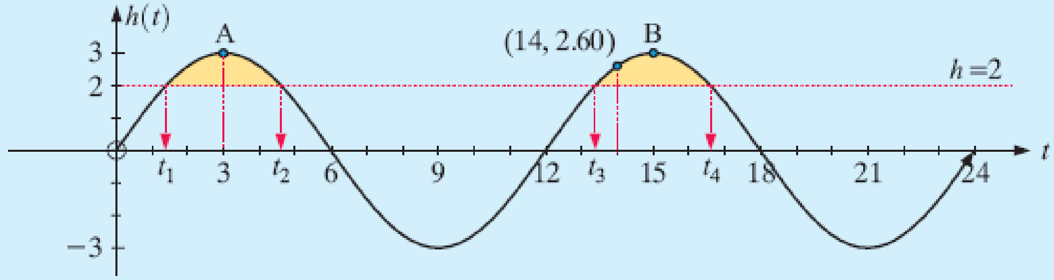
d If a ship can cross the harbour provided the tide is at least 2 m above mean sea level, when is crossing possible on January 24?

a $h(t) = 3\sin\left(\frac{\pi t}{6}\right)$ has period $= \frac{2\pi}{\frac{\pi}{6}} = 2\pi \times \frac{6}{\pi} = 12$ hours and $h(0) = 0$



c At 2 pm, $t = 14$ and $h(14) = 3\sin\left(\frac{14\pi}{6}\right) \approx 2.60$ (3 sig figs)
So the tide is 2.6 m above the mean.

d



We need to solve $h(t) = 2$ i.e., $3\sin\left(\frac{\pi t}{6}\right) = 2$.

Using a graphics calculator with $Y_1 = 3\sin\left(\frac{\pi X}{6}\right)$ and $Y_2 = 2$

we obtain $t_1 = 1.39$, $t_2 = 4.61$, $t_3 = 13.39$, $t_4 = 16.61$

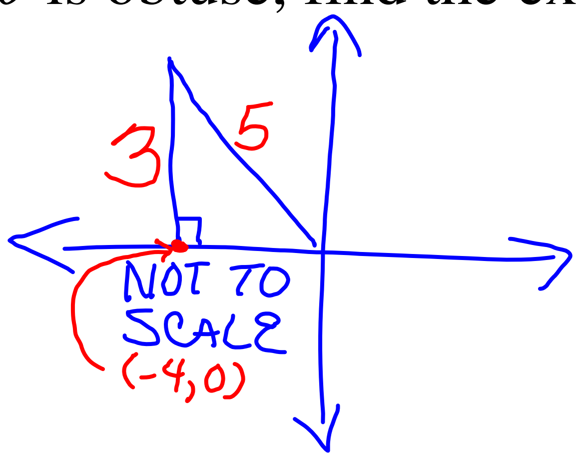
Now 1.39 hours = 1 hour 23 minutes, etc.

$\therefore$ can cross between 1:23 am and 4:37 am or 1:23 pm and 4:37 pm.

Here are a few non-calculator questions

Given that $\sin\theta = \dfrac{3}{5}$ and θ is obtuse, find the exact values of

(a) $\cos\theta = -\dfrac{4}{5}$



(b) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\dfrac{3}{5}\right)\left(-\dfrac{4}{5}\right)$

$$= -\frac{24}{25}$$

(c) $\cos 2\theta = 1 - 2\sin^2\theta$

$$= 1 - 2\left(\frac{3}{5}\right)^2$$

$$= \frac{7}{25}$$

Another non-calculator question

Find $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

let $A = \frac{\pi}{3}$

$B = \frac{\pi}{4}$

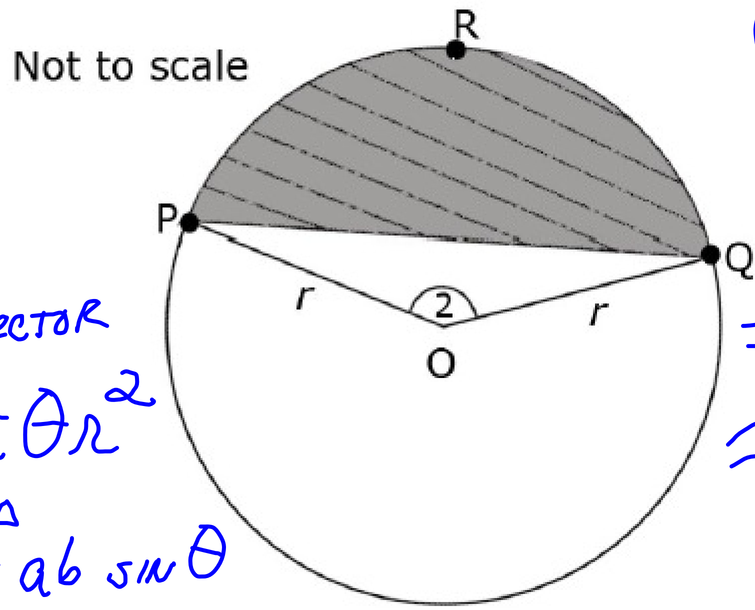
$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

How about a sector problem?!

Radians are used in this question.



$\text{Area}_{\text{SHADED}} =$

$\text{Area}_{\text{sector}} - \text{Area}_{\Delta}$

$= \frac{1}{2}(2)(5)^2 - \frac{1}{2}(5)(5)\sin 2$

$\approx 13.6 \text{ cm}^2$

$\text{Area}_{\text{SECTOR}}$

$= \frac{1}{2}\theta r^2$

Area_{Δ}

$= \frac{1}{2}ab\sin\theta$

a) Given that the perimeter of *OPRQ* is 20 cm, show that r=5cm.

b) Hence find the shaded area of the shape.

a) Perimeter of $OPR\overset{\frown}{Q}$

$= 2r + \text{length of } \overset{\frown}{PRQ}$

$= 2r + \theta r$

$20\text{cm} = 2r + 2r$

$20\text{cm} = 4r$

$5\text{cm} = r$

Solve algebraically but you can use your g.d.c. for the final solutions $\cos 2\theta = 2\cos^2\theta - 1$

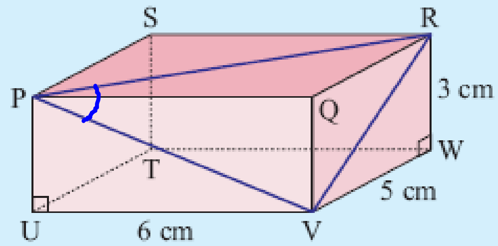
$$\frac{1}{1+\tan^2\theta} + 3\cos(2\theta) = 1 \qquad \text{For } 0 \le \theta \le 2\pi$$

$$\cos^2\theta + 3[2\cos^2\theta - 1] = 1$$
$$\cos^2\theta + 6\cos^2\theta - 3 = 1$$
$$7\cos^2\theta = 4$$
$$\cos^2\theta = \frac{4}{7}$$
$$\cos\theta = \pm\sqrt{\frac{4}{7}}$$
$$\theta \approx .714$$
$$\approx 2.43$$
$$\approx .714 + \pi$$
$$\approx 2.43 + \pi$$

And now for the problems that give me headaches:

Example 7

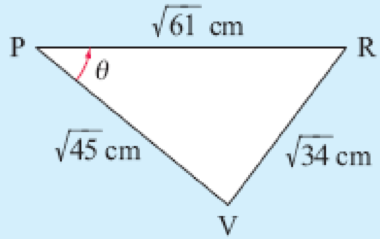
Find the measure of angle RPV.



In ΔRVW, $RV = \sqrt{5^2+3^2} = \sqrt{34}$ cm. {Pythagoras}

In ΔPUV, $PV = \sqrt{6^2+3^2} = \sqrt{45}$ cm. {Pythagoras}

In ΔPQR, $PR = \sqrt{6^2+5^2} = \sqrt{61}$ cm. {Pythagoras}



$$\cos\theta = \frac{(\sqrt{61})^2 + (\sqrt{45})^2 - (\sqrt{34})^2}{2\sqrt{61}\sqrt{45}}$$

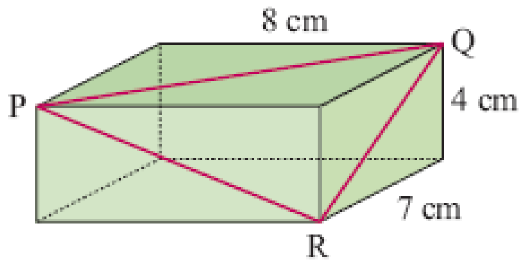
$$= \frac{61+45-34}{2\sqrt{61}\sqrt{45}}$$

$$= \frac{72}{2\sqrt{61}\sqrt{45}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{36}{\sqrt{61}\sqrt{45}}\right) \approx 46.6^o$$

$\therefore$ angle RPV measures about 46.6^o.

Now you try:



Find the measure of angle PQR in the rectangular box shown.