

Graphing and transforming functions

This is a big topic in the IB curriculum!

Please see page 120

We will be considering the following types of functions:
Linear, quadratic, cubic, absolute value, exponential,
logarithmic, and reciprocal

[You might want to review transformations of
trigonometric functions too.]

What we want to focus on:

Intercepts, extreme values, domain and range, asymptotes

Invariant points [points that do not move]

Please do the Transformation Investigation handout by
yourself. [Just you and your g.d.c.]

The answers are in the powerpoint

Ibhltransfcurves.ppt

I would like for us to try some. These are from Chapter 6
in our textbook.

Summary of graphical transformations on $y = f(x)$

- ▶ For $y = f(x) + b$, the effect of b is to **translate** the graph **vertically** through b units.
 - If $b > 0$ it moves **upwards**.
 - If $b < 0$ it moves **downwards**.
- ▶ For $y = f(x - a)$, the effect of a is to **translate** the graph **horizontally** through a units.
 - If $a > 0$ it moves to the **right**.
 - If $a < 0$ it moves to the **left**.
- ▶ For $y = f(x - a) + b$, the graph is translated horizontally a units and vertically b units. We say it is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$.

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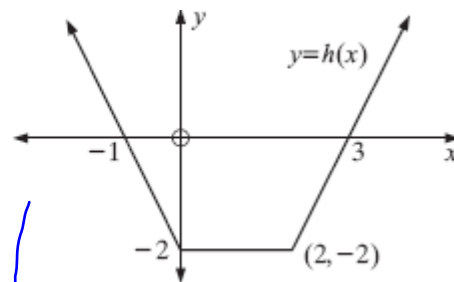
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- ▶ For $y = pf(x)$, $p > 0$, the effect of p is to **vertically stretch** the graph by a factor of p .
 - If $p > 1$ it moves points of $y = f(x)$ **further away** from the x -axis.
 - If $0 < p < 1$ it moves points of $y = f(x)$ **closer** to the x -axis.
- ▶ For $y = f(kx)$, $k > 0$, the effect of k is to **horizontally compress** the graph by a factor of k .
 - If $k > 1$ it moves points of $y = f(x)$ **closer** to the y -axis.
 - If $0 < k < 1$ it moves points of $y = f(x)$ **further away** from the y -axis.
- ▶ For $y = -f(x)$, the effect is to **reflect** $y = f(x)$ **in the x -axis**.
- ▶ For $y = f(-x)$, the effect is to **reflect** $y = f(x)$ **in the y -axis**.

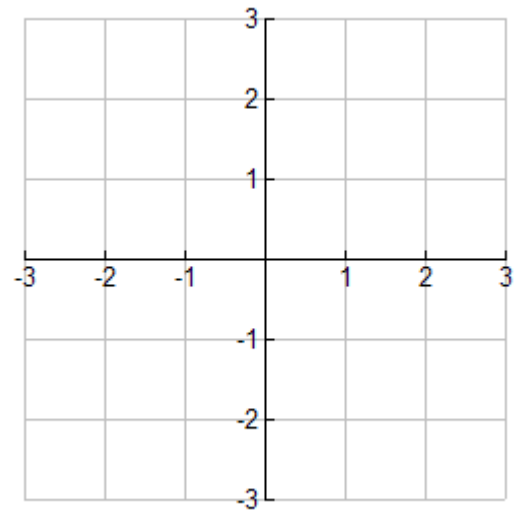
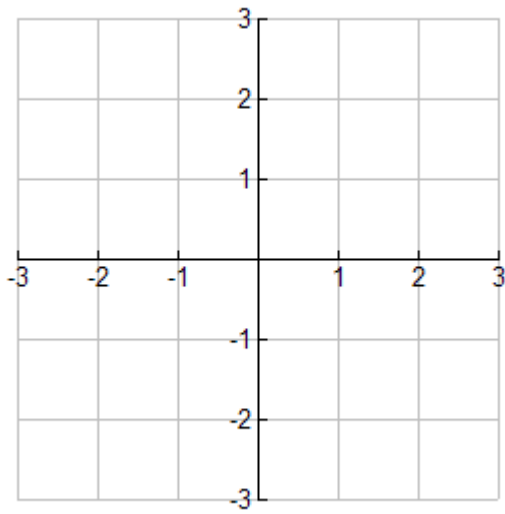
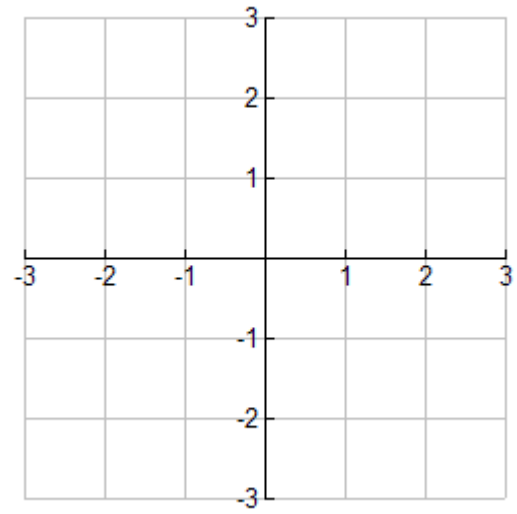
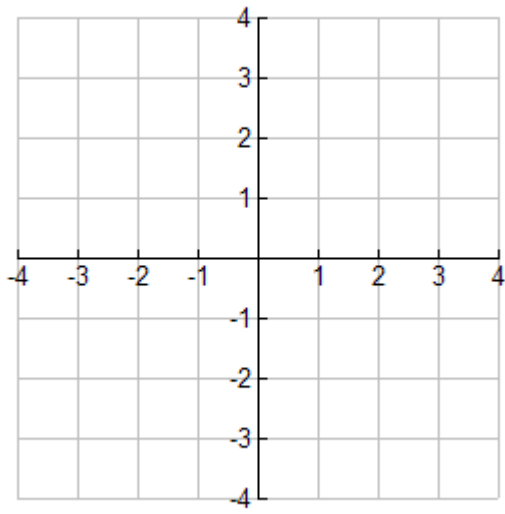
Note: Stretching by a factor of $\frac{1}{p}$ is equivalent to compressing by a factor of p .
 For example, $y = \frac{1}{3}f(x)$ indicates that $y = f(x)$ is vertically stretched by a factor of $\frac{1}{3}$, or compressed by a factor of 3.
 Likewise, $y = f(\frac{1}{4}x)$ indicates that $y = f(x)$ is horizontally compressed by a factor of $\frac{1}{4}$, or stretched by a factor of 4.

For the graph of $y = h(x)$ given, draw sketches of:

- a** $y = h(x) + 1$
- b** $y = \frac{1}{2}h(x)$
- c** $y = h(-x)$
- d** $y = h(\frac{x}{2})$



- (a) VERTICALLY UP | $(2, -2) \rightarrow (2, -1)$
- (b) VERTICAL STRETCH of $\frac{1}{2}$ (MOVE CLOSER) | $(2, -2) \rightarrow (2, -1)$
y-values shrink
- (c) REFLECT y-AXIS | $(2, -2) \rightarrow (-2, -2)$
- (d) HORIZONTAL STRETCH by FACTOR of $\frac{1}{2}$ | COMPRESSED

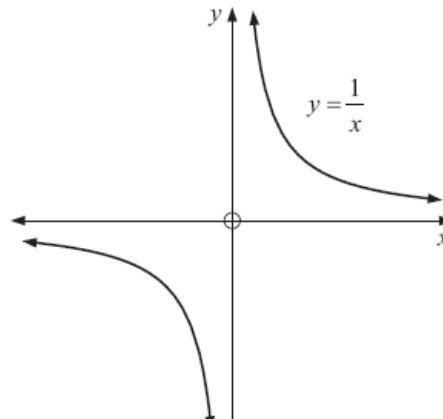


Any function $x \mapsto \frac{ax+b}{cx+d}$, $x \neq -\frac{d}{c}$ where a, b, c and d are constants is called a **simple rational function**.

These functions are characterised by the presence of both a **horizontal asymptote (HA)** and a **vertical asymptote (VA)**.

Any graph of a simple rational function can be obtained from the reciprocal function $x \mapsto \frac{1}{x}$ by a combination of transformations including:

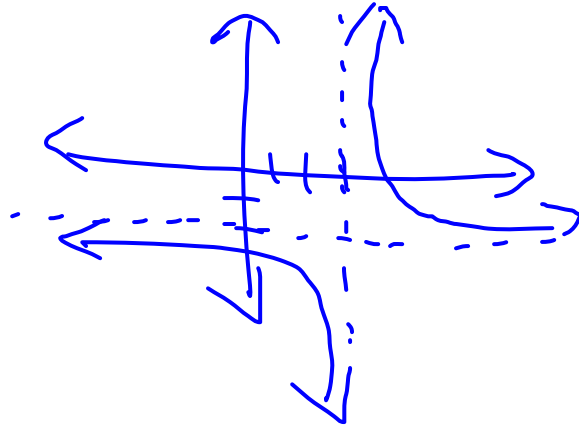
- a **translation**
(vertical and/or horizontal)
- **stretches and compressions**
(vertical and/or horizontal)



- a** Find the function $y = g(x)$ that results when transforming the reciprocal function, $x \mapsto \frac{1}{x}$ by: a vertical stretch with factor 2 then a horizontal compression with factor 3, then a translation of $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$.
- b** Find the asymptotes of each function found in **a**.
- c** Is the function found in **a** a self inverse function? Explain.

(a) $\frac{1}{x}$ becomes $2\left(\frac{1}{x}\right)$
 $\frac{2}{x}$ becomes $\frac{2}{3x}$
 replace x with $x-3$
 $\frac{2}{3x}$ becomes $\frac{2}{3(x-3)}$

$\frac{2}{3x-9} - 2$
 $x = 3$
 $y = -2$
 VERTICAL ASYMPTOTE
 HORIZONTAL ASYMPTOTE



Transformations involving the “modulus” That’s IB-speak for absolute value

Draw the graph of $f(x) = 3x(x - 2)$ and on the same axes draw the graphs of:

a $y = |f(x)|$

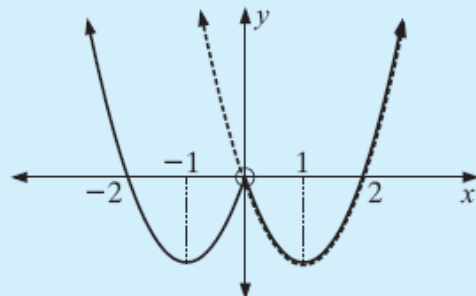
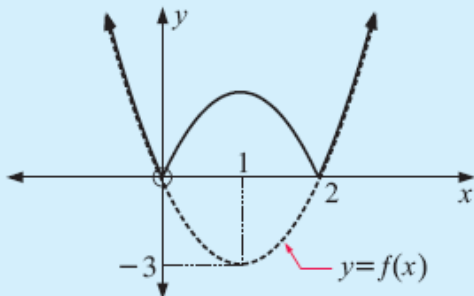
b $y = f(|x|)$

a $y = |f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$

b $y = f(|x|) = \begin{cases} f(x) & \text{if } x \geq 0 \\ f(-x) & \text{if } x < 0 \end{cases}$

This means the graph is unchanged for $f(x) \geq 0$, reflected in the x -axis for $f(x) < 0$.

This means the graph is unchanged if $x \geq 0$, reflected in the y -axis if $x < 0$.



To try:

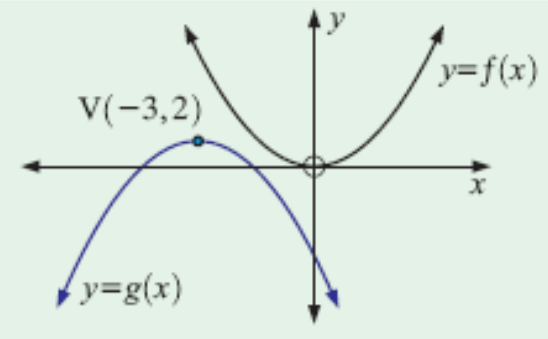
- a** If $f(x) = x + 2$, find the equation of the function F obtained by *stretching* the function f *vertically* by a factor of 2 and compressing the function *horizontally* by a factor of 2, followed by a translation of $\frac{1}{2}$ horizontally and -3 vertically.
- b**
- i** Check that the point $(1, 3)$ remains invariant under the transformation described in part **a**.
 - ii** What happens to the points $(0, 2)$ and $(-1, 1)$ under the transformation given in part **a**?
 - iii** Show that the points in **ii** lie on the graph of $y = F(x)$.

$f(x) \rightarrow F(x)$
 $x+2$ becomes $2(x+2) = 2x+4$
 $2x+4$ becomes $2(2x)+4 = 4x+4$
 $4x+4$ becomes $4(x-\frac{1}{2})+4 = 4x+2$
 $4x+2$ becomes $4x+2-3 = 4x-1$

$f(1) = 3$ $F(1) = 3$ 😊
 $(0, 2) \rightarrow (\frac{1}{2}, 1)$ $(-1, 1) \rightarrow (0, -1)$
 $F(0) = -1$ ✓
 $F(\frac{1}{2}) = 1$ ✓

The graph of $f(x) = x^2$ is transformed to the graph of $g(x)$ by a reflection and a translation as illustrated in the diagram alongside.

Find the equation of $y = g(x)$ in the form $ax^2 + bx + c$.



reflexion
 $x \rightarrow -x$

DISCUSSION

THE GRAPHICAL CONNECTION BETWEEN

$$y = f(x) \quad \text{AND} \quad y = \frac{1}{f(x)}$$



True or false? Discuss:

- The zeros of $f(x)$ become VA values of $\frac{1}{f(x)}$ and the VA values of $f(x)$ become the zeros of $\frac{1}{f(x)}$.
- Maximum values of $f(x)$ become minimum values of $\frac{1}{f(x)}$ and the minimum values of $f(x)$ become maximum values of $\frac{1}{f(x)}$.
- When $f(x) > 0$, $\frac{1}{f(x)} > 0$ also and when $f(x) < 0$, $\frac{1}{f(x)} < 0$ also.
- When $f(x) \rightarrow 0$, $\frac{1}{f(x)} \rightarrow \pm\infty$ and when $\frac{1}{f(x)} \rightarrow 0$, $f(x) \rightarrow \pm\infty$.