

The last word about transformations [okay, maybe not the last]

Transformation matrices

To transform a function using a matrix $\begin{pmatrix} a \\ b \end{pmatrix}$, just replace the x with $x - a$ and replace the y with $y - b$

For example:

Let $f(x) = \frac{2}{3x}$ and do the translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$$y - (-2) = \frac{2}{3(x-3)}$$

The pretty version would be: $y = \frac{2}{3x-9} - 2$

What transformation occurred?

The graph moved 3 to the right and down 2

Let's just try a few

Consider the function $y = 3x^2 + \pi e$

Translate using the transformation matrix $\begin{pmatrix} 9 \\ 23 \end{pmatrix}$ MOVES 9 RIGHT
UP 23

becomes $y = 3x^2 + \pi e$

$$y - 23 = 3(x - 9)^2 + \pi e$$

Last one [Yay!]

This one is from our textbook and I think that they did a good job of it.

Consider $f(x) = \frac{4x + 3}{x - 2}$

(a) Find the asymptotes of $y = f(x)$

$x = 2$ VERTICAL ASYMPTOTE

$$\lim_{x \rightarrow \infty} f(x) = 4$$

$y = 4$ HORIZONTAL ASYMPTOTE

(b) Find the intercepts

$$f(x) = \frac{4x+3}{x-2}$$

$$f(0) = \frac{3}{-2} \quad (0, -\frac{3}{2})$$

$$0 = \frac{4x+3}{x-2} \rightarrow x = -\frac{3}{4}$$

$(-\frac{3}{4}, 0)$

(c) Discuss the behaviour of the function near its asymptotes

They mean find the limits

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

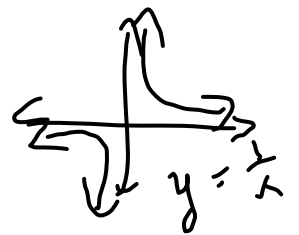
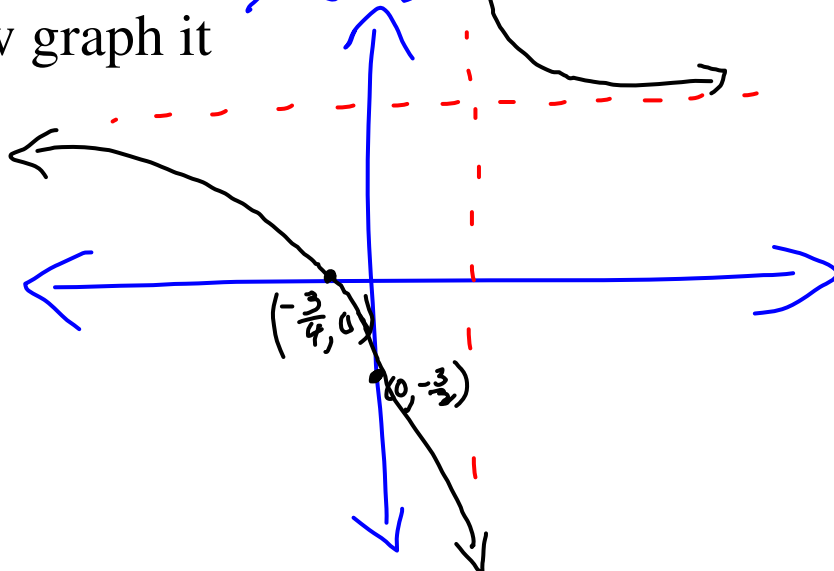
$$\lim_{x \rightarrow 2^+} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = 4$$

$$\lim_{x \rightarrow \infty} f(x) = 4$$

FROM below
FROM above

(d) Now graph it



(e) Describe the transformation which move $x \mapsto \frac{1}{x}$

$$\begin{aligned} f(x) &= \frac{4x + 3}{x - 2} \\ &= \frac{4(x-2) + 11}{x-2} \\ &= \frac{4(x-2)}{x-2} + \frac{11}{x-2} \\ &= 4 + \frac{11}{x-2} \end{aligned}$$

$$\frac{1}{x} \mapsto \frac{11}{x} \quad \text{v.s.}$$

TRAN. MATRIX $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$