# Statistical Distributions – Discrete Random variables

A random variable represents, in number form, the possible outcomes for some random experiment.

A discrete random variable, $X$, has possible values $x = 0, 1, 2, 3, 4, ...$

Example: the number of colleges that a high school student applies to or the number of cracked eggs in a carton

In mathematics, "discrete" means "countable"

A continuous random variable, $X$, has all possible values in some interval.

Example: the age of students could be in the interval $11 \le x \le 21$

Sometimes, "continuous" implies "measurable"

Let's classify:

| Random variable | Continuous | Discrete |
|---|---|---|
| Number of bears in the woods | | ✓ |
| Height of a bear | ✓ | |
| Grade on a "maths" test | | ✓ |
| Number of students who took a "maths" test | | ✓ |
| Weight of a student who took a "maths" test | ✓ | |
| Number of hairs on a student who took a "maths" test | | ✓ |
| Number of coins in a student's pocket | | ✓ |

For ANY random variable, there is a probability distribution associated with it.

Standard notation:

$P(X = x)$

Here's what a typical problem looks like:

A "maths" student is playing a game where a turn consists of throwing a dice and then moving the number of squares on the playing board equal to the score on the dice.

In this case, $X =$ the number of squares moved in a turn

$P(X = 3)$ means "the probability that they move 3 squares on the playing board

For a fair die our Probability Distribution will look like:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | = 1 |

What would the Probability Distribution look like for the following:

Two dice are thrown and the number, $X$ , is the sum of the dice which are equal to the number of squares that are moved.

How many different sums are possible? 11

Based on our work, what are they probabilities?

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P(X=x)$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

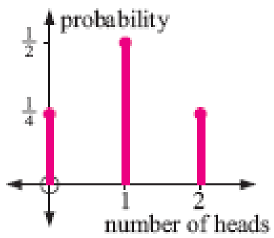
Notice that the sum of $P(X=x)$ is equal to one.

For each random variable there is a probability distribution, $p_i$, where $0 \le p_i \le 1$ AND $\sum_{i=1}^{n} p_i = 1$.

We can represent our probability distribution of a discrete random variable by using a table, or a graph, or in function form.

For example, when tossing two coins, the random variable $X$ could be 0 heads, 1 head, or 2 heads, i.e., $X = 0$, 1 or 2. The associated probability distribution is $p_0 = \frac{1}{4}$, $p_1 = \frac{1}{2}$, and $p_2 = \frac{1}{4}$ with graph:

$P(X=0)$
$=$ NO HEADS

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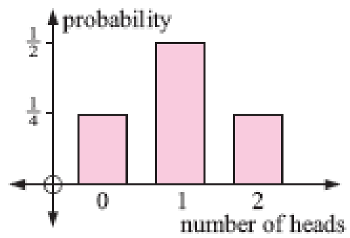



See page 729 in our textbook:

**Example 1**

A supermarket has three checkout points A, B and C. A government inspector checks for accuracy of the weighing scales at each checkout. If a weighing scale is accurate then yes (Y) is recorded, and if not, no (N). Suppose the random variable $X$ is the number of accurate weighing scales at the supermarket.

**a** List the possible outcomes.

**b** Describe using $x$ the events of there being:

**i** one accurate scale **ii** at least one accurate scale.

**a** Possible outcomes:

| A | B | C | $x$ |
|---|---|---|---|
| N | N | N | 0 |
| Y | N | N | 1 |
| N | Y | N | 1 |
| N | N | Y | 1 |
| N | Y | Y | 2 |
| Y | N | Y | 2 |
| Y | Y | N | 2 |
| Y | Y | Y | 3 |

**b** **i** $x = 1$

**ii** $x = 1$, 2 or 3

$P(X=1) = \frac{3}{8}$

$P(X=1,2,\text{or } 3) = 1 - P(X=0)$
$= 1 - \frac{1}{8}$
$= \frac{7}{8}$

Let's try #4 on page 729:

Consider tossing three coins simultaneously. The random variable under consideration is the number of heads that could result.

**a** List the possible values of $x$. $x = 0, 1, 2, 3$

**b** Tabulate the possible outcomes and the corresponding values of $x$.

**c** Find the values of $P(X = x)$, the probability of each $x$ value occurring.

**d** Graph the probability distribution $P(X = x)$ against $x$ as a probability histogram.

SAMPLE SPACE = {TTT, HTT, THT, TTH, THH, HTH, HHT, HHH}

$P(X=0) = \frac{1}{8}$ $\quad$ $P(X=2) = \frac{3}{8}$

$P(X=1) = \frac{3}{8}$ $\quad$ $P(X=3) = \frac{1}{8}$

**Example 2**

A magazine store recorded the number of magazines purchased by its customers in one week. 23% purchased one magazine, 38% purchased two, 21% purchased three, 13% purchased four, and 5% purchased five.

**a** What is the random variable?

**b** Make a probability table for the random variable.

**c** Graph the probability distribution using a spike graph.

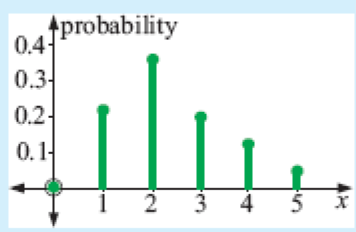
**a** The random variable $X$ is the number of magazines sold.

So, $x = 0, 1, 2, 3, 4$ or $5$.

**b**

| $x_i$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $p_i$ | 0.00 | 0.23 | 0.38 | 0.21 | 0.13 | 0.05 |

**c**




But probability distribution can also look like this:

(a) $P(X = x) = \dfrac{x^2 + 1}{34}$ for $x = 1, 2, 3, 4$

Let's make a table:

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{2}{34}$ | $\frac{5}{34}$ | $\frac{10}{34}$ | $\frac{17}{34}$ |

Now let's find $\sum P(i)$ $= \frac{1}{34}(2+5+10+17)$

$= 1$

(b) $P(X = x) = C^3_x \, (0.6)^x \, (0.4)^{3-x}$ for $x = 0, 1, 2, 3$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | .064 | .288 | .432 | .216 |

Now let's find $\sum P(i)$ $= 1$

Let's try #8 on the top of page 732

Electrical components are produced and packed into boxes of 10. It is known that 10% of the components may be faulty. The random variable $X$ denotes the number of faulty items in the box and has a probability distribution of

$P(x) = C_x^{10}(0.04)^x(0.96)^{10-x}$, for $x =$0, 1, 2, 3, …, 10

(a) Find the probability that a randomly selected box will contain NO faulty components.

$$P(x=0) = \binom{10}{0}(0.04)^0(0.96)^{10-0}$$

$$\approx .6648$$

(b) Find the probability that a randomly selected box will contain at least one faulty component.

$$P(\text{at least 1 faulty}) = 1 - P(x=0)$$
$$= 1 - .6648$$
$$= .3352$$

Now onto "Expectation"

Expectation

If there are $n$ members of a sample and the probability of an event occurring is $p$ for each member, then the expectation of the occurrence of that event is $n \bullet p$

Example:

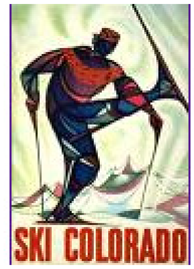
If we were to toss a coin 100 times, then how many times would we 'expect" to get "tails"?

$n = 100;\ p = \frac{1}{2}$ So, our expectation is 50.

Here is a nice Colorado-themed example:

During the snow season, there is a $\frac{4}{7}$ probability of snow falling on any particular day. If Skippy skis for two months, then how many days could he expect to get "fresh powder"?

assume $n = 60$ days



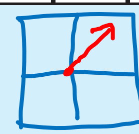

$n = 60$

$p = \frac{4}{7}$ $\quad P(E) = 60\left(\frac{4}{7}\right)$

**Example 6**

In a game of chance, a player spins a square spinner labelled 1, 2, 3, 4. The player wins the amount of money shown in the table alongside, depending on which number comes up.

| *Number* | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| *Winnings* | \$1 | \$2 | \$5 | \$8 |



Determine:

**a** the expected return for one spin of the spinner

**b** whether you would recommend playing this game if it costs \$5 for one game.

**a** As each number is equally likely, the probability for each number is $\frac{1}{4}$

$\therefore$ expected return $= \frac{1}{4} \times 1 + \frac{1}{4} \times 2 + \frac{1}{4} \times 5 + \frac{1}{4} \times 8 = \$4$.

**b** As the expected return of \$4 is less than the cost of \$5 to play the game, you would not recommend that a person play the game.

"Expectation by Formulae"

The expectation of a random variable $X$ is defined by $E(X) = \sum x_i\, p_i$ where $x_i$ represents particular outcomes, $p_i$ represents the probability of $x_i$ occurring, and $X$ is the random variable.

Note: $E(x) = \mu$ in some textbooks

Here is a great example:

A roulette wheel is numbered 1 through 37. In some particular gaming hall, if the ball lands on 10, then you win \$10; if the ball lands on a number ending in 5, you win \$5; otherwise, you win nothing. The amount, $X$, that you win is a random variable. If the roulette wheel is fair, $X$ takes the values 10, 5, and 0. Let's find the probabilities:

$$P(X = 10) = \frac{1}{37}$$

$$P(X = 5) = \frac{4}{37}$$

$$P(X = 0) = \frac{32}{37}$$

Now let's make a probability distribution:

| | | | |
|---|---|---|---|
| $x$ | 0 | 5 | 10 |
| $P(X = x)$ | $\frac{32}{37}$ | $\frac{4}{37}$ | $\frac{1}{37}$ |

Now to find the expectation or $E(X)$:

$E(X) = 0\left(\frac{32}{37}\right) + 5\left(\frac{4}{37}\right) + 10\left(\frac{1}{37}\right)$

$\approx \$.81$

Let's consider #7 on page 734

A person rolls a normal six-sided die and wins the number of dollars shown on the face.

(a) How much does the person expect to win for one roll of the die?

$E(x) = \frac{1}{6}(1+2+3+4+5+6)$

$= \$3.50$

(b) If it costs \$4 to play the game, would you advise the person to play several games? NO

A person plays a game with a pair of coins. If two heads appear then \$10 is won. If a head and a tail appear then \$3 is won. If two tails appear then \$5 is lost.

**a** How much would a person expect to win playing this game once?

**b** If the organiser of the game is allowed to make an average of \$1 per game, how much should be charged to play the game once? \$3.75

| Result | Win |
|---|---|
| HH | \$10 |
| TH HT | \$3 |
| TT | -\$5 |

$$E(X) = \frac{1}{4}(10) + \frac{1}{2}(3) + \frac{1}{4}(-5)$$

$$= \$2.75$$