SEQUENCES AND SERIES REVISION
[NOT CALCULUS, BUT ALGEBRA]

HERE IS WHAT THE OFFICIAL FORMULA PACKET PROVIDES

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
	The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$
	The sum of an infinite geometric sequence	$S = \frac{u_1}{1-r}$, $\lvert r \rvert < 1$

Our textbook has a lot of good examples in Chapter Two.

$\{u_n\}$ is **arithmetic** $\Leftrightarrow$ $u_{n+1} - u_n = d$ for all positive integers n where d is a constant (the **common difference**).

$n \in Z^+$

$\{2n+1\} =$ 3, 5, 7, 9, …

How to find the arithmetic mean

If a, b and c are any consecutive terms of an arithmetic sequence then

$$\begin{aligned} b - a &= c - b \qquad \{\text{equating common differences}\} \\ \therefore \quad 2b &= a + c \\ \therefore \quad b &= \frac{a+c}{2} \end{aligned}$$

So, the middle term is the **arithmetic mean** of the terms on either side of it.

Consider the sequence 2, 9, 16, 23, 30, ...

a Show that the sequence is arithmetic.

b Find the formula for the general term u_n.

c Find the100th term of the sequence.

d Is **i** 828 **ii** 2341 a member of the sequence?

(a) $9-2=7$
$16-9=7$
$23-16=7$

$d=7$

(b) $u_n = u_1 + (n-1)d$
$u_n = 2 + (n-1)(7)$
$u_n = 7n-5$

(c) $u_{100} = 7(100)-5$
$= 695$

(d) $828 = 7n-5$
$833 = 7n$
$119 = n$

$2341 = 7n-5$
$2346 = 7n$
$335\frac{1}{7} = n$

How to find the general term:

Find the general term u_n for an arithmetic sequence with $u_3 = 8$ and $u_8 = -17$.

$u_3 = 8$

$8 = u_1 + 2d$

$u_8 = -17$

$-17 = u_1 + 7d$

$u_n = 18 + -5(n-1)$

$$\left.\begin{array}{l} 8 = u_1 + 2d \\ -17 = u_1 + 7d \end{array}\right\}$$

$25 = -5d$

$-5 = d$

$u_1 = 18$

Filling out a sequence:

Insert four numbers between 3 and 12 so that all six numbers are in arithmetic sequence.

$u_1, u_2, u_3, u_4, u_5, u_6$

$3, 3+d, 3+2d, 3+3d, 3+4d, \underset{3+5d}{12}$

$12 = 3 + 5d$

$1.8 = d$

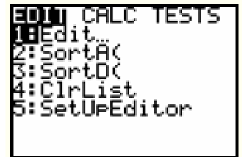
$3, 4.8, 6.6, 8.4, 10.2, 12$

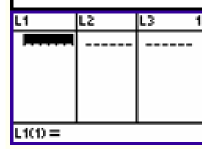
Did you notice that the formula for the general term is just a linear function?

A way to check your formula:

1) Click on the STAT button of your TI 84
2) Select "Edit".

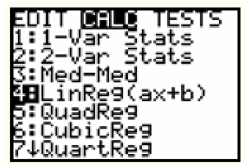
Here you will see vertical columns with l_1 (list 1), L_2(list 2) etc.. These columns are where you type in your ordered paris, your x and y values. X values goes in l_1 and y values go into l_2.





3) Scroll RIGHT to "CALC" and press the enter buton

4) Scroll down to "LinReg(ax +b) "



5) Then press L1, "," and , "L2" (to get 'l1' press '2nd' then the '1' key) and ","

Graphing calculator screen should now show:

LinReg(ax +b) L1 , L2 ,

6) The next step will create the graph. Press "Vars" scroll to 'Y-Vars' and select "Function". Choose Y1 (you are now storing this linear regression in the y1 graph so that you can graph the actual regression.

Let's try this out!

Now onto Geometric Sequences

$\{u_n\}$ is **geometric** $\Leftrightarrow$ $\dfrac{u_{n+1}}{u_n} = r$ for all positive integers n

where r is a **constant** called the **common ratio**.

If a, b and c are any consecutive terms of a geometric sequence then $\dfrac{b}{a} = \dfrac{c}{b}$.

$\therefore$ $b^2 = ac$ and so $b = \pm\sqrt{ac}$ where $\sqrt{ac}$ is the **geometric mean** of a and c.

THE GENERAL TERM

Suppose the first term of a geometric sequence is u_1 and the common ratio is r.

Then $u_2 = u_1 r$, $u_3 = u_1 r^2$, $u_4 = u_1 r^3$, and so on.

Hence $u_n = u_1 r^{n-1}$

The power of r is one less than the subscript.

So, for a **geometric sequence** with **first term** u_1 and **common ratio** r, the **general term** (or **nth term**) is $u_n = u_1 r^{n-1}$.

For the sequence $8, 4, 2, 1, \frac{1}{2}, ...$

a Show that the sequence is geometric.

b Find the general term u_n.

c Hence, find the 12th term as a fraction.

(a) $\dfrac{4}{8} = \dfrac{1}{2}$ $\quad \dfrac{1}{2} = \dfrac{1}{2}$

$\dfrac{2}{4} = \dfrac{1}{2}$ $\quad \dfrac{\frac{1}{2}}{1} = \dfrac{1}{2}$ $\quad r = \dfrac{1}{2}$

(b) $u_n = u_1 r^{n-1}$

$u_n = 8\left(\dfrac{1}{2}\right)^{n-1}$

(c) $u_{12} = 8\left(\dfrac{1}{2}\right)^{11}$

$= 2^3(2^{-1})^{11} = 2^{-8}$

$k-1,\ 2k$ and $21-k$ are consecutive terms of a geometric sequence. Find k.

$$\frac{2k}{k-1} = \frac{21-k}{2k}$$

$$4k^2 = (21-k)(k-1)$$

$$5k^2 - 22k + 21 = 0$$

$$(5k-7)(k-3) = 0$$

$k = 3$ $(r=3)$

$2, 6, 18, \ldots$

$k = \frac{7}{5}$ $(r=7)$

$\frac{2}{5}, \frac{14}{5}, \frac{98}{5}, \ldots$

A geometric sequence has $u_2 = -6$ and $u_5 = 162$. Find its general term.

$u_2 = u_1 r$

$u_5 = u_1 r^4$

$$\frac{162}{-6} = \frac{\cancel{u_1} r^4}{\cancel{u_1} r}$$

$-6 = u_1(-3)$

$2 = u_1$

$-27 = r^3$

$-3 = r$

$$u_n = 2(-3)^{n-1}$$