ZZZZZZZZZZZZZZZZZZZZZZZZZZ

To obtain a standard normal distribution [also called a "z-distribution", you need to convert your values to a Z value where $Z = \frac{X - \mu}{\sigma}$

Let's look at the investigation on page 754

Suppose a random variable X is normally distributed with mean μ and standard deviation σ. For each value of X we can calculate a Z-value using the transformation: $Z = \frac{X - \mu}{\sigma}$

What is the mean and standard deviation of a Z-distribution?

On your calculator use the X values:

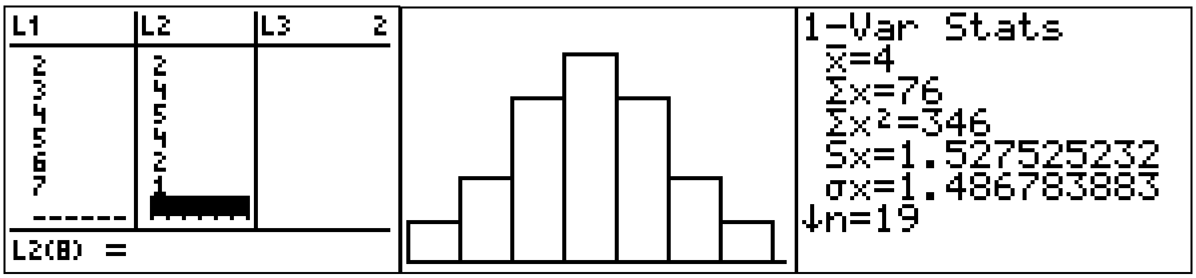
1,2,2,3,3,3,3,4,4,4,4,4,5,5,5,5,6,6,7

Draw a graph

Find the mean and standard deviation

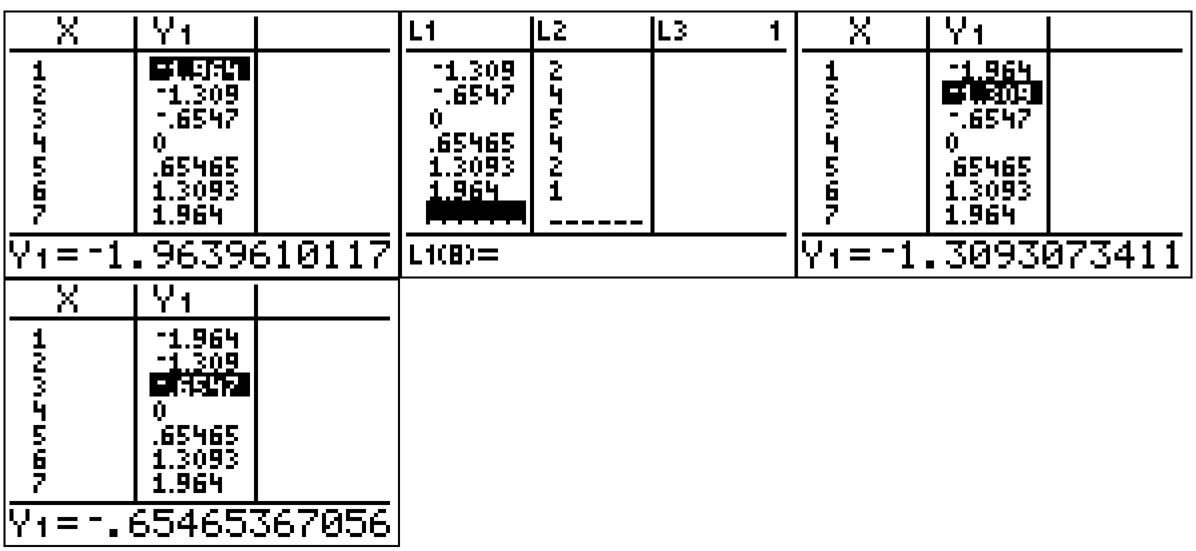
Use the transformation $Z = \frac{X - \mu}{\sigma}$ to convert the X-values to Z-values

Now find the mean and standard deviation of the distribution of the Z values.



$$Z = \frac{X - \mu}{\sigma} \qquad \mu = 4, \sigma = 1.527525232$$

$$Z(1) = \frac{1 - 4}{1.527525232}$$



After "z-ifying" my *x*-values I get that our new mean = 0 and our new standard deviation is 1. [Hey! It's as close to one as I need]

```
1-Var Stats
 x̄=0
 Σx=0
 Σx²=18
 Sx=.9999999998
 σx=.9733285266
↓n=19
```

What was the point of this? Well, for the normal X-distribution we have the probability density function of: $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

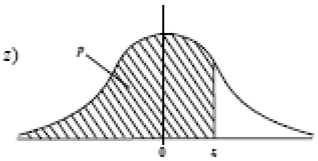
Now let $x=z,\ \mu=0,\ \sigma=1$ to get the probability density function for the Z-distribution

$$f(z)=\frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}} \text{ for } -\infty < z < \infty$$

Our IB official formula packet has a statistical table that starts on page 13. It looks like this:

Inverse normal probabilities (topic 6.11)

$p = P(Z \le z)$



p	0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0176	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2379	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2534	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2872	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4262	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5331	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6434	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6714
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031

And back in the day, before graphing calculators, we used this table to find $P(Z \le z)$

[No one liked it. If they said they did, they were not remembering well.]

Now most people enjoy the convenience of just using a graphing calculator.

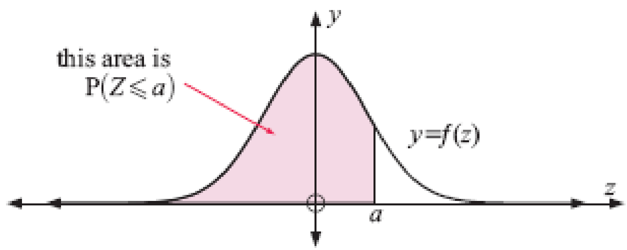
CALCULATING PROBABILITIES USING THE Z-DISTRIBUTION

Since z is continuous,

$$P(Z \leqslant a) = P(Z < a)$$

and $P(Z \leqslant a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, dz.$

The table of curve areas on **page 824** enables us to find $P(Z \leqslant a)$.



USING A GRAPHICS CALCULATOR TO FIND PROBABILITIES

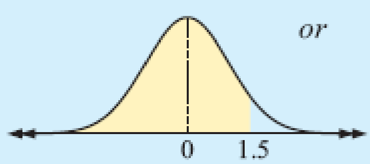
For a **TI-83**:

To find	$P(Z \leqslant a)$ or $P(Z < a)$	use	normalcdf($-$E99, a).
To find	$P(Z \geqslant a)$ or $P(Z > a)$	use	normalcdf(a, E99).
To find	$P(a \leqslant Z \leqslant b)$ or $P(a < Z < b)$	use	normalcdf(a, b).

If Z is a standard normal variable, find:

a $P(Z \leqslant 1.5)$ **b** $P(Z > 0.84)$ **c** $P(-0.41 \leqslant Z \leqslant 0.67)$

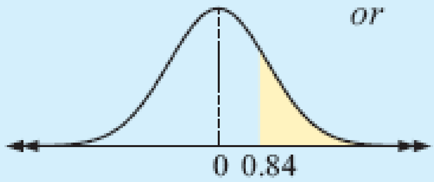
a $P(Z \leqslant 1.5)$
≈ 0.933



or $P(Z \leqslant 1.5)$
$=$ normalcdf($-$E99, 1.5)
≈ 0.933

But we can also get this value from our statistical table. Look on page 13 for 1.5 and then go to the "0" column. Read the number.

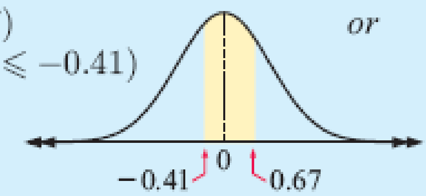
$P(Z > 0.84)$
$= 1 - P(Z \leqslant 0.84)$
$\approx 1 - 0.79954$
≈ 0.200



or $P(Z > 0.84)$
$=$ normalcdf(0.84, E99)
≈ 0.200

The table only gives Z < z for you would have needed to rewrite. [It helped me to always draw the shaded region to figure out what I needed.

c $P(-0.41 \leqslant Z \leqslant 0.67)$
$= P(Z \leqslant 0.67) - P(Z \leqslant -0.41)$
$\approx 0.7486 - 0.3409$
≈ 0.408



or $P(-0.41 \leqslant Z \leqslant 0.67)$
$=$ normalcdf(−0.41, 0.67)
≈ 0.408

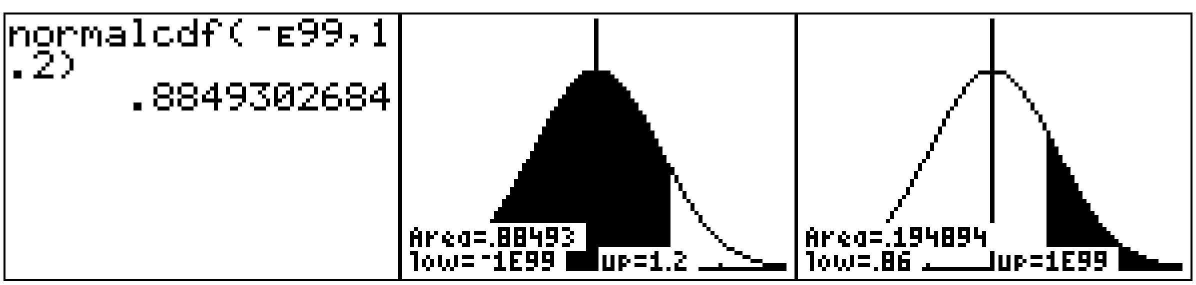
See page 757 #2 and #3

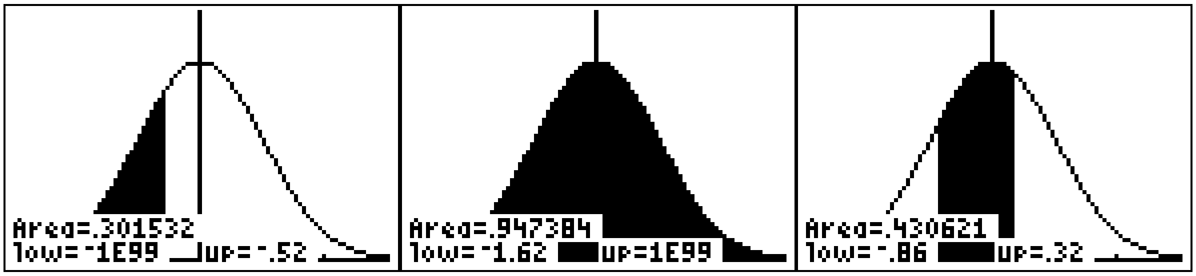
2 If Z has standard normal distribution, find *using tables* and a sketch:

a $P(Z \leqslant 1.2)$ **b** $P(Z \geqslant 0.86)$ **c** $P(Z \leqslant -0.52)$
d $P(Z \geqslant -1.62)$ **e** $P(-0.86 \leqslant Z \leqslant 0.32)$

3 If Z has standard normal distribution, find *using technology*:

a $P(Z \geqslant 0.837)$ **b** $P(Z \leqslant 0.0614)$ **c** $P(Z \geqslant -0.876)$
d $P(-0.3862 \leqslant Z \leqslant 0.2506)$ **e** $P(-2.367 \leqslant Z \leqslant -0.6503)$






```
1-normalcdf(⁻E99
,.837)
      .2012962287
```

```
normalcdf(⁻E99,.
0614)
      .5244797446
```

```
1-normalcdf(⁻E99
,⁻.876)
      .8094850257
```

```
normalcdf(⁻.3862
,.2506)
      .2492639307
```

```
normalcdf(⁻2.367
,⁻.6503)
       .248782709
```