

Probability and Statistics Problems to Ponder

Calculator

A random sample is drawn from a large population contains the following data:

6.2, 7.8, 12.1, 9.7, 5.2, 14.8, 16.2, 3.7

Calculate an unbiased estimate of

(a) The population mean; $\bar{x} \approx 9.46$

(b) The population variance $\text{VAR}(x) \approx 20.8$

Non-calculator

Box A contains 6 red balls and 2 green balls.

Box B contains 4 red balls and 3 green balls. A

fair cubical die with faces numbered 1, 2, 3, 4,

5, 6 is thrown. If an even number is obtained, a

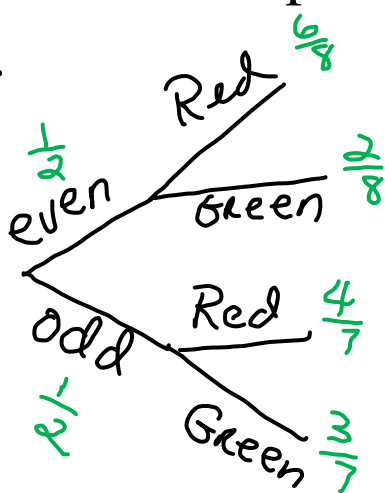
ball is selected from Box A; if an odd number is

obtained, a ball is selected from Box B.

(a) Calculate the probability that the ball selected was red.

(b) Given that the ball selected was red, calculate the probability that it came from Box

B.



$$P(\text{Red}) = \left(\frac{1}{2}\right)\left(\frac{6}{8}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{7}\right) = \frac{37}{56}$$

$$P(B|R) = \frac{\frac{2}{7}}{\frac{37}{56}} = \frac{16}{37}$$

Calculator

A random variable X is normally distributed with mean μ and variance σ^2

If $P(X > 6.2) = 0.9474$ and $P(X < 9.8) = 0.6368$, calculate the value of μ and σ

All we need is $Z = \frac{X - \mu}{\sigma}$

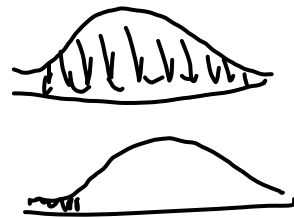
$$\frac{6.2 - \mu}{\sigma} = -1.62$$

$$\frac{9.8 - \mu}{\sigma} = .35$$

$$\mu = 9.16$$

$$\sigma = 1.83$$

$$\text{Var}(x) = 3.34$$



Calculator

Let X be a normal random variable with mean 25 and variance 4. Find $P(|X - 25| < 3)$

IF $\text{VAR}(X) = 4$, then $\sigma = 2$

$$\begin{aligned} & P(|X - 25| < 3) \\ &= P(22 < X < 28) \\ &\approx .866 \end{aligned}$$

Calculator

The probability density function of the random variable X is defined by

$$f(x) = \begin{cases} \frac{\pi}{2} - 2x \arctan x, & 0 \leq x \leq 1 \\ 0 & \textit{otherwise} \end{cases}$$

The value of a is such that $P(X < a) = 0.75$

(i) Show that a satisfies the equation

$$a(2\pi + 4) = 3 + 4(a^2 + 1)\arctan a$$

[You need to show all of your integration work]

(ii) Find the value of a

