More Calculus Review Questions to Ponder

Paper One Style [non-calculator]

Easy warm-up [Show all working]

$$\int_0^1 \left(e^{2x} + \frac{1}{x+1} \right) dx$$

$$= \frac{1}{2}e^{2x} + \ln(x+1) \Big|_0^1$$

$$= \left(\frac{1}{2}e^2 + \ln 2\right) - \left(\frac{1}{2}\right)$$

Calculate the exact value [w/o calculator]

$$\int_1^e x^2 \ln x \, dx$$

$u = \ln x \quad V = \frac{x^3}{3}$

$du = \frac{1}{x}dx \quad dV = x^2 dx$

$$= \frac{1}{3}x^3 \ln x \Big|_1^e - \int_1^e \left(\frac{x^3}{3}\right)\left(\frac{1}{x}\right)dx$$

$$= \frac{1}{3}x^3 \ln x \Big|_1^e - \frac{1}{3}\int_1^e x^2 dx$$

$$= \frac{1}{3}x^3 \ln x \Big|_1^e - \frac{1}{9}x^3 \Big|_1^e \quad = \frac{2}{9}e^3 + \frac{1}{9}$$

Consider the graph of $f\ [-5, 1] \mapsto \mathbb{R}$, where $f(x) = x^3 + 6x^2 + 9x + 20$.

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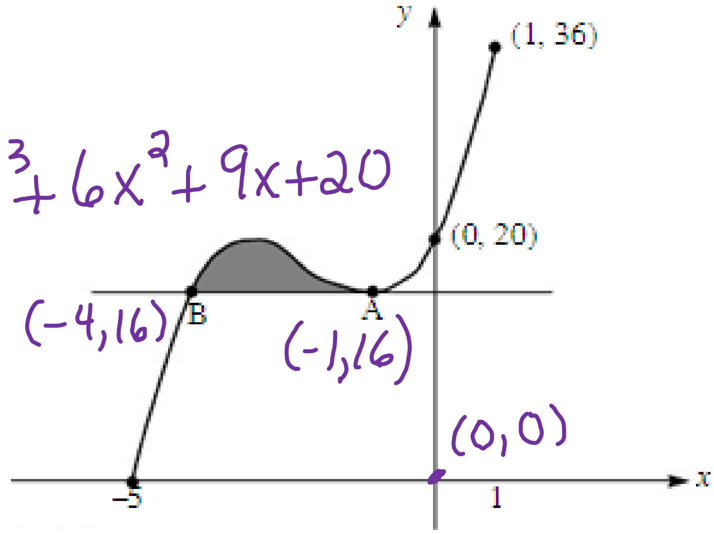


(a) (i) Find $f'(x)$.

(ii) Find the two values of x where the tangent to the graph of $f(x)$ is horizontal.

(b) (i) Expand $(x+1)^2(x+4)$.

(ii) Find where the tangent line to $f(x)$ at A extends to meet at B.

(iii) Find the area of the shaded region shown.

$f'(x) = 3x^2 + 12x + 9$

$f'(x) = 0$

$0 = 3(x+3)(x+1)$

$x = -1$ and $x = -3$

$(x+4)(x+1)^2 = x^3 + 6x^2 + 9x + 4$

A $(-1, 16)$

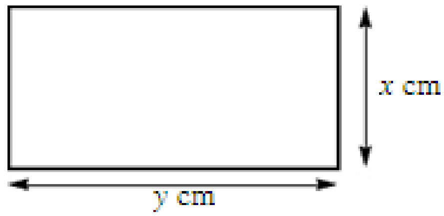
$16 = x^3 + 6x^2 + 9x + 20$

$0 = x^3 + 6x^2 + 9x + 4$ EGADS!

Hence $B = (-4, 16)$

$\text{area}_{\text{(SHADED)}} = \int_{-4}^{-1} [f(x) - 16]\,dx = 6.75$

The rectangle of area A cm^2 and dimensions x cm by y cm has a constant perimeter of 20 cm.



(a) Show that (i) $y = 10 - x$.

(ii) $A = 10x - x^2$

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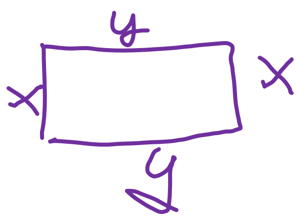
(b) (i) Find $\frac{dA}{dx}$

(ii) Hence find the maximum area and justify your answer.



$P = 2x + 2y$

$20 = 2x + 2y$

$10 = x + y$

$10 - x = y$

$A = xy$

$A = x(10 - x)$

$A = 10x - x^2$

$\frac{d}{dx} A = \frac{d}{dx}(10x - x^2)$

$\frac{dA}{dx} = 10 - 2x$

$\frac{dA}{dx} = 0$ when $x = 5$

at $x = 5$ $\frac{dA}{dx}$ changes from positive to negative values. Hence A has a max when $x = 5$ cm

max Area

$= (5cm)(5cm)$

$= 25 cm^2$

Consider the integral $I_n = \int_0^1 \frac{t^n}{(1+t)^n}\, dt$

Show that $\frac{t}{t+1} = 1 - \frac{1}{t+1}$

Evaluate I_1

Differentiate $\ln\left(t^2+2t+1\right)$

Show that $\dfrac{t^2}{\left(1+t^2\right)}=1-\dfrac{2t+1}{(t+1)^2}$

Hence evaluate I_2