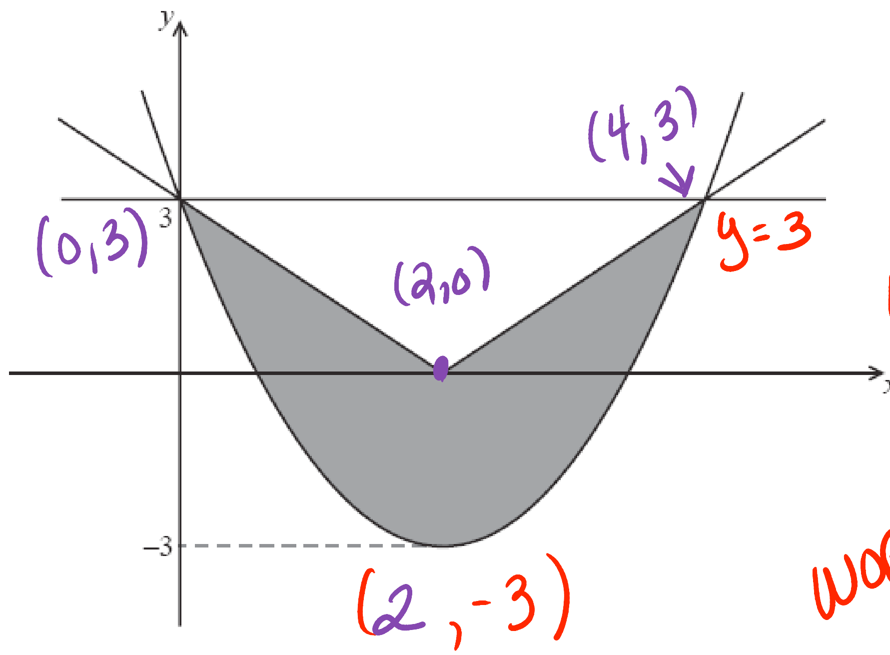
Calculus Potpourri #3

This one is calculator

The diagram below shows the graphs of $y = \left|\frac{3}{2}x - 3\right|$, $y = 3$ and a quadratic function, that all intersect in the same two points.



$y = \left|\frac{3}{2}x - 3\right|$

worth 8 MARKS!

Given that the minimum value of the quadratic function is -3, find an expression for the area of the shaded region in the form $\int_0^t (ax^2 + bx + c)\,\mathrm{d}x$, where the constants a, b, c and t are to be determined. (Note: The integral does not need to be evaluated.)

$\left|\frac{3}{2}x - 3\right| = 0$ when $x = 2$

PARABOLA goes (0,3) $\quad y = p(x-2)^2 - 3$

Parabola $\quad y = \frac{3}{2}(x-2)^2 - 3$

Area$_{\text{SHADED}}$ $= 2\int_0^2 \left[\left(3 - \frac{3}{2}x\right) - \left(\frac{3}{2}x^2 - 6x + 3\right)\right] dx$

$= \int_0^2 (-3x^2 + 9x)\, dx$

A big calculator question

A body is moving through a liquid so that its acceleration can be expressed as

$$\left(-\frac{v^2}{200}-32\right)\mathrm{m\,s^{-2}},$$

$$\frac{dv}{dt}=\left(\frac{-v^2}{200}-32\right)$$

where $v\ \mathrm{m\,s^{-1}}$ is the velocity of the body at time t seconds.

The initial velocity of the body was known to be $40\ \mathrm{m\,s^{-1}}$. $v(0)=40\ \frac{m}{sec}$

(a) Show that the time taken, T seconds, for the body to slow to $V\ \mathrm{m\,s^{-1}}$ is given by

$$T=200\int_V^{40}\frac{1}{v^2+80^2}\,dv.$$ *[4 marks]*

(b) (i) Explain why acceleration can be expressed as $v\frac{dv}{ds}$, where s is displacement, in metres, of the body at time t seconds.

(ii) **Hence** find a similar integral to that shown in part (a) for the distance, S metres, travelled as the body slows to $V\ \mathrm{m\,s^{-1}}$. *[7 marks]*

(c) **Hence**, using parts (a) and (b), find the distance travelled and the time taken until the body momentarily comes to rest. *[3 marks]*

(a)

$$\frac{dv}{dt}=\frac{-v^2}{200}-32$$

$$\frac{dv}{dt}=\frac{-v^2-6400}{200}$$

$$\int_{40}^{V}-\frac{200}{v^2+80^2}\,dv=\int_0^T dt$$

$$200\int_V^{40}\frac{1}{v^2+80^2}\,dv=T$$

(b) $$a = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt}$$ ✓ Hey! That's ✓

$$= v\,\frac{dv}{ds}$$

OR DIMENSIONAL ANALYSIS

$$v\,\frac{dv}{ds} = \frac{-v^2 - 80^2}{200}$$

$$\int_0^{S} ds = \int_{40}^{V} -\frac{200v}{v^2+80^2}\,dv$$

$$S = 200\int_{V}^{40} \frac{v}{v^2+80^2}\,dv$$

(c) letting $V = 0$

$$\text{distance} = 200\int_0^{40} \frac{v}{v^2+80^2}\,dv \approx 22.3\text{m}$$

$$\text{Time} = 200\int_0^{40} \frac{1}{v^2+80^2}\,dv \approx 1.16 \text{ seconds}$$

Now let’s stroll down memory lane and think about converging and diverging series

I like to think about the biblical character, Moses and the parting of the seas

P p-series

A alternating series

R ratio test

T telescoping series

I integral test

N n^{th} term

G geometric series

C comparison test