

CALCULATOR OKAY

(i) Find the Maclaurin series of the function

$f(x) = \ln(1 + \sin x)$ up to and including the term in x^4

(ii) (a)

Use the trapezium rule with three ordinates to

estimate the value of the integral $\int_{0.5}^1 \sin(x^2) dx$

Give your answer correct to six decimal places

(b) Calculate an upper bound for the error in this estimate

(c) Find the value of this integral to six decimal places. Verify that the error is less than your upper bound

(iii) Consider the equation $2 \sin x - x = 0$

(a) By drawing a suitable sketch, determine the number of real roots of this equation

(b) Find the value, α , of the root between 1 and 2, giving your answer to the accuracy displayed on your calculator

(c) Using the Newton-Raphson method, with $x_0 = 2$, find successive approximations x_1 and x_2 to this root of the equation. Give your answers to the accuracy displayed on your calculator.

(d) You are given that successive approximations satisfy, approximately, the equation $(x_{n+1} - \alpha) = k(x_n - \alpha)^N$, ($n \geq 0$) where k is a constant and N is a positive integer. By putting $n = 0$ and $n = 1$ and substituting your results from part (c), determine the value of N . Hence state the order of convergence of the Newton-Raphson method.

