More DiffEQ

May 2007 Paper 3

Use integration by parts to show that

$$\int \sin x \cos x e^{-\sin x} dx = -e^{-\sin x}(1+\sin x)+C$$

$I = uv - \int v\, du$

$u = \sin x$

$du = \cos x\, dx$

$v = -e^{-\sin x}$

$dv = \cos x e^{-\sin x} dx$

$= -\sin x e^{-\sin x} + \int \cos x e^{-\sin x} dx$

$= -\sin x e^{-\sin x} - e^{-\sin x} + C$

$= -e^{-\sin x}(1+\sin x) + C$

Consider the differential equation

$$\frac{dy}{dx} - y\cos x = \sin x \cos x$$

(i) Find an integrating factor call it $u(x)$

$u(x) = e^{\int -\cos x\, dx}$

$= e^{-\sin x}$ $(+c)$

we know we need this!

(ii) Solve the differential equation, given that $y = -2$ when $x = 0$. Give your answer in the form $y = f(x)$

$$y = \frac{1}{u(x)} \int Q(x)\, u(x)\, dx$$

$$y = \frac{1}{e^{-\sin x}} \int \sin x \cos x \, e^{-\sin x} \, dx$$

$$= \frac{1}{e^{-\sin x}} \left[-e^{-\sin x}(1+\sin x) + c \right]$$

Given $(0, -2)$

$$-2 = \frac{1}{e^{0}} \left[-e^{0}(1+\sin 0) + c \right]$$

Hence, $c = -1$

AND SO IT CAME TO PASS

$$y = -1 - \sin x - e^{\sin x}$$

May 2006 Paper 3

(a) Show that $\int \tan x \, dx = \ln \sec x + C$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$-\int \frac{1}{u} du$

$$= -\ln|\cos x| + C$$

$$= \ln|\sec x| + C$$

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(b) Hence find an integrating factor for solving the differential equation $\dfrac{dy}{dx} + y \tan x = \sec x$

$$u(x) = e^{\int \tan x \, dx}$$

$$= e^{\ln|\sec x|}$$

$$= \sec x$$

(c) Solve this differential equation given that $y = 2$ when $x = 0$. Give your answer in the form $y = f(x)$

$$y = \frac{1}{u(x)} \int Q(x)\,u(x)\,dx$$

$$= \frac{1}{\sec x} \int \sec x \sec x \, dx$$

$$= \frac{1}{\sec x} (\tan x + c)$$

$(0,2)$

$2 = 1 \, (0 + c)$ thus $c = 2$

$$y = \sin x + 2\cos x$$

How about another one so we can be experts!

Given that $y = 4$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} - y\tan x = 4\sec^2 x$$

$$u(x) = e^{\int -\tan x \, dx}$$

$$= e^{\ln|\cos x| (+c)}$$

$$= \cos x$$

$$y = \frac{1}{u(x)} \int Q(x)\, u(x)\, dx$$

$$= \frac{1}{\cos x} \int 4\sec^2 x \cos x \, dx$$

$$= \frac{1}{\cos x} \int 4 \sec x \, dx$$

$$= \frac{1}{\cos x} \left(4\ln(\sec x + \tan x) + c\right)$$

$(0,4)$

$$4 = 1\left(4\ln(1+0) + c\right)$$

Hence $c = 4$

Thus, $y = \sec x \left(4\ln(\sec x + \tan x) + 4\right)$

A potpourri of Diff EQ

Find the general solution of the differential equation

$$\frac{dy}{dx} - \frac{y}{x} = 6x^2 + 10x$$

giving y in terms of x in your answer.

Find the general solution of the differential equation:

$$5\frac{dy}{dx} + 5y = 0$$