Significance of Standard Deviation

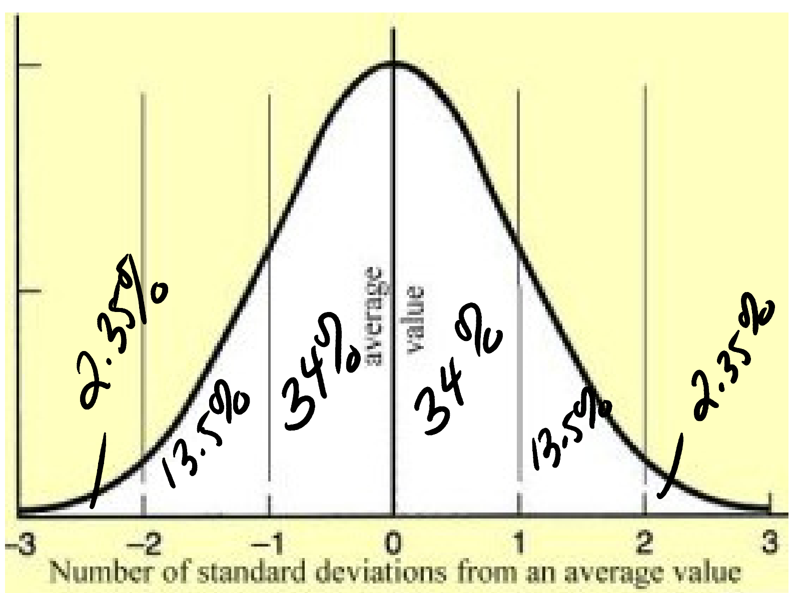
By now, you are familiar with how some standardized exams use a "Bell Curve".

If a large sample from a typical bell curve is taken, what percentage of the data values would like

between $\overline{x} - s$ and $\overline{x} + s$?

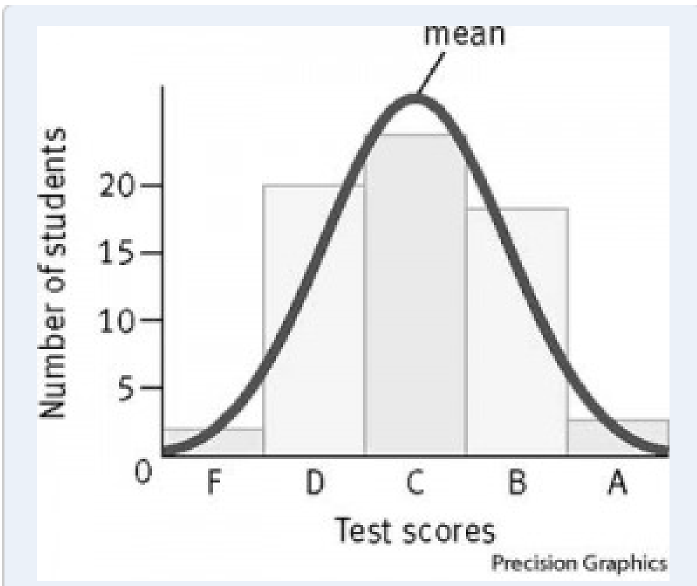
Interpretation: What percentage of values lies within one standard deviation of the mean?

- approximately **68%** of the population will measure between **1** standard deviation either side of the mean
- approximately **95%** of the population will measure between **2** standard deviations either side of the mean
- approximately **99.7%** of the population will measure between **3** standard deviations either side of the mean.



If you have a college class where the professor uses a bell curve, then that means that approximately 68% of the class would receive a grade of “C”.

From Google Images:



Notice the beautiful symmetry!

Example 15

A sample of 200 cans of peaches was taken from a warehouse and the contents of each can measured for net weight. The sample mean was 486 g with standard deviation 6.2 g. What proportion of the cans might lie within:

a 1 standard deviation from the mean **b** 3 standard deviations from the mean?

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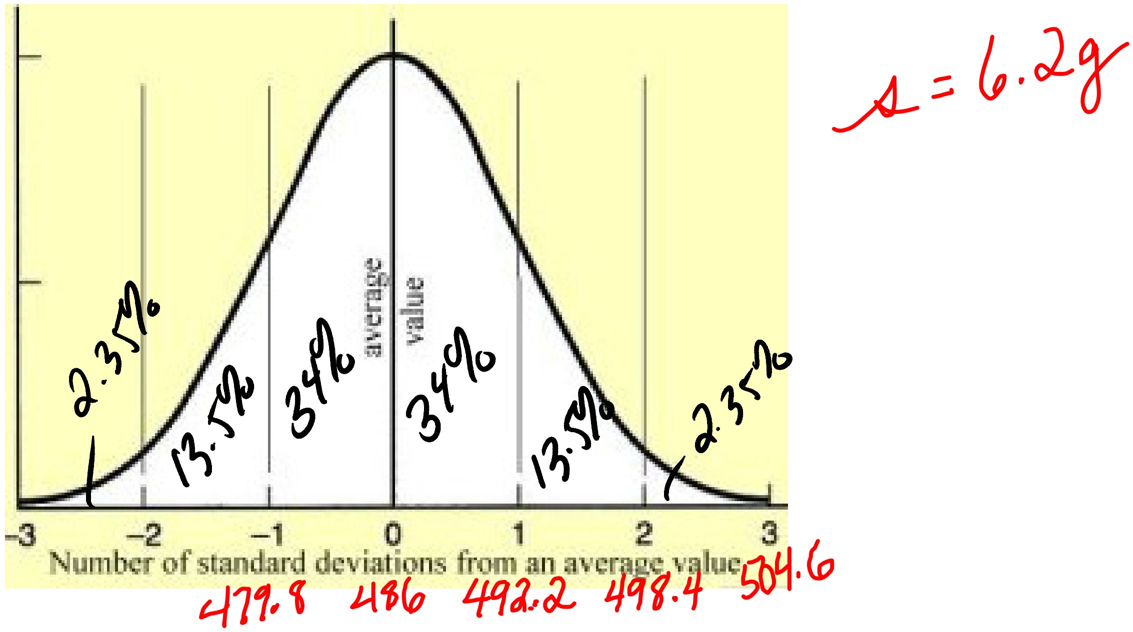
a About 68% of the cans would be expected to have contents between
486 ± 6.2 g i.e., 479.8 g and 492.2 g.

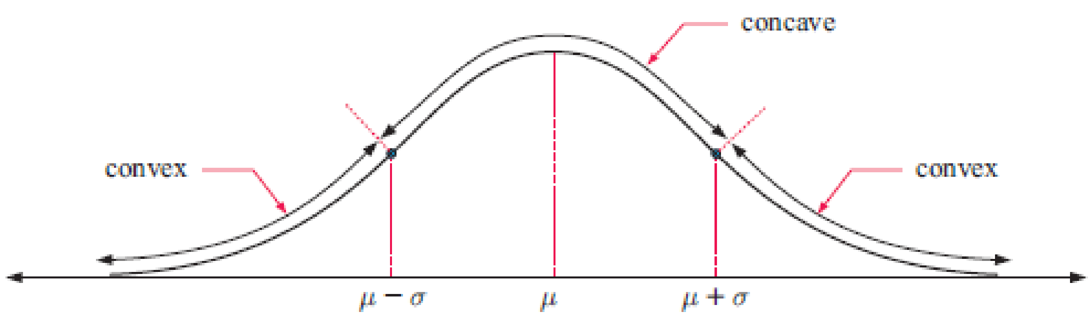
b Nearly all of the cans would be expected to have contents between
$486 \pm 3 \times 6.2$ g i.e., 467.4 and 504.6 g.



Our textbook has a good explanation about how in "real life", the normal curve is rarely found.

THE NORMAL CURVE

The smooth curve that models normally distributed data is asymptotic to the horizontal axis, so in theory there are no limits within which all the members of the population will fall.



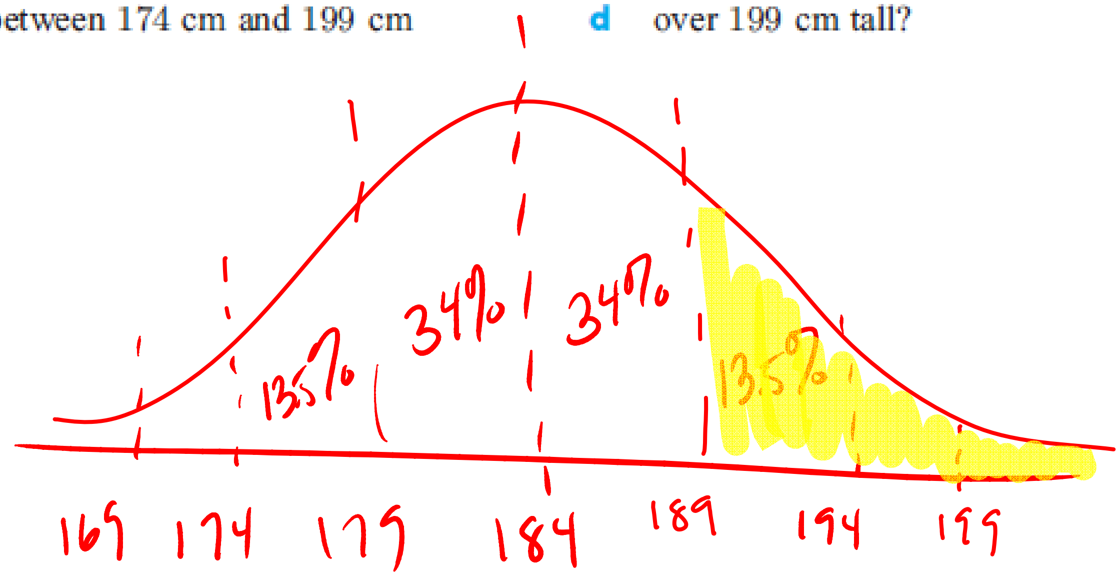
In practice, however, it is rare to find data outside of 3 standard deviations from the mean, and *exceptionally* rare to find data beyond 5 standard deviations from the mean.

Note that the position of 1 standard deviation either side of the mean corresponds to the point where the normal curve changes from a concave to a convex curve.

Let’s do some easy problems.

The mean height of players in a basketball competition is 184 cm. If the standard deviation is 5 cm, what percentage of them are likely to be:

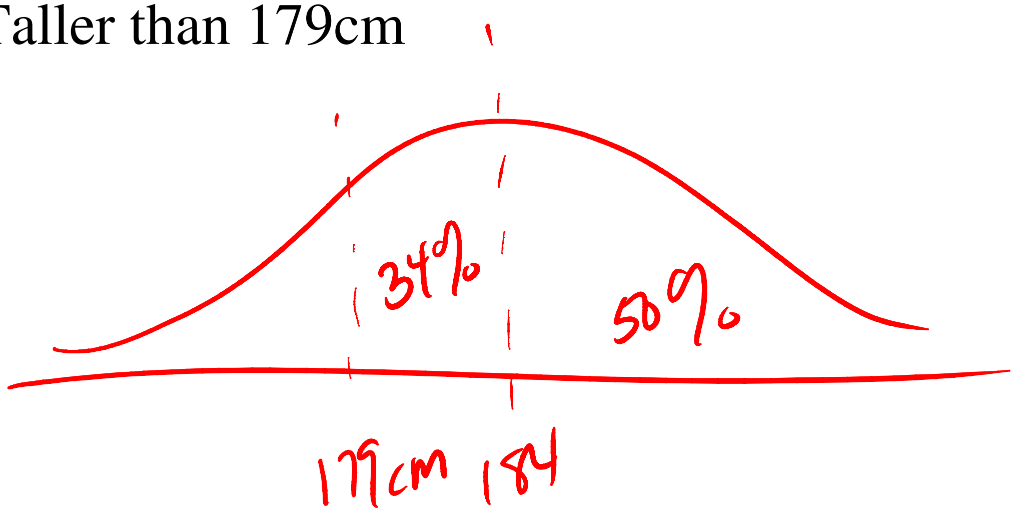
- **a** taller than 189 cm
- **b** taller than 179 cm
- **c** between 174 cm and 199 cm
- **d** over 199 cm tall?



Taller than 189cm = 100% − (50% + 34%)
= 16%

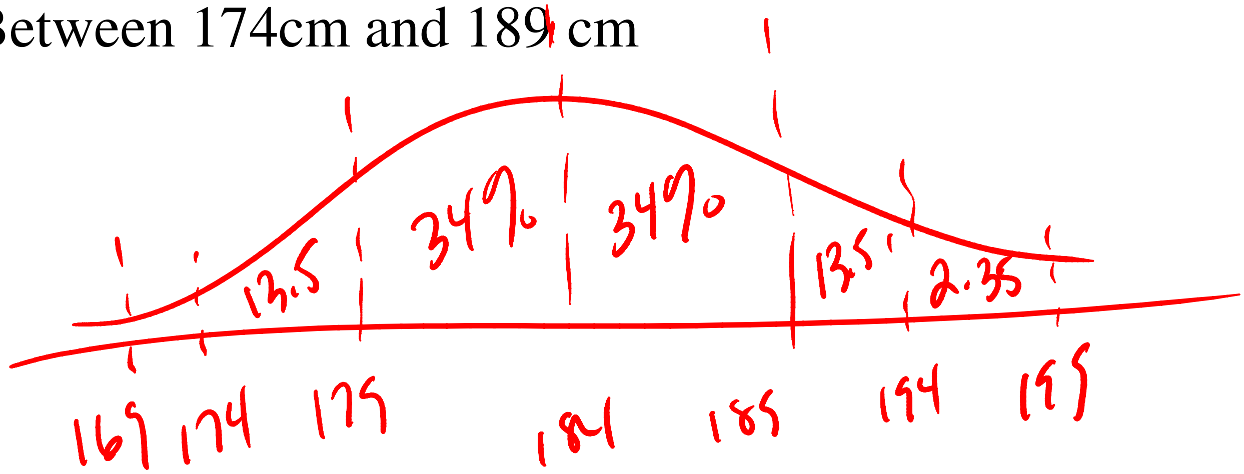
(b)

Taller than 179cm



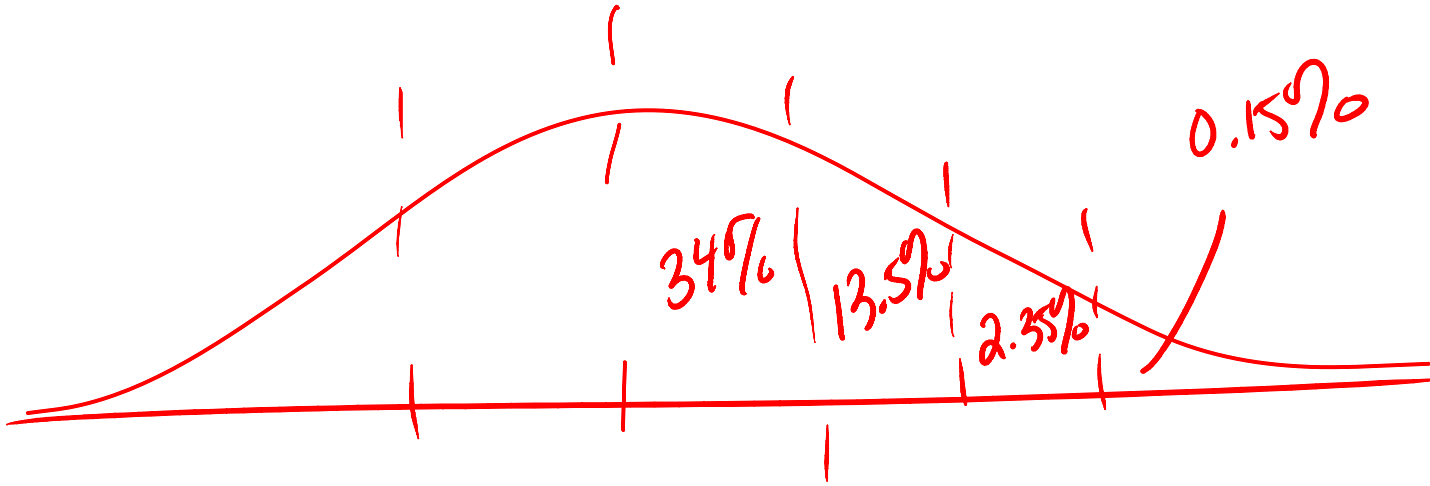
(c)

Between 174cm and 189 cm



$$2(34\%) + 2(13.5\%) + 2.35\% \approx 97.4\%$$

(d) Over 199cm tall $\approx .15\%$



Now you try #2, 3, and 4 on page 499

2 The mean average rainfall of Claudona for August is 48 mm with a standard deviation of 6 mm. Over a 20 year period, how many times would you expect there to be less than 42 mm of rainfall during August in Claudona?

3 Two hundred lifesavers competed in a swimming race. The mean time was 10 minutes 30 seconds. The standard deviation was 15 seconds. Find the number of competitors who probably:

- **a** took longer than 11 minutes
- **b** took less than 10 minutes 15 seconds
- **c** completed the race in a time between 10 min 15 sec and 10 min 45 sec.

4 The weights of babies born at Prince Louis Maternity Hospital last year averaged 3.0 kg with a standard deviation of 200 grams. If there were 545 babies born at this hospital last year, estimate the number that weighed:

- **a** less than 3.2 kg
- **b** between 2.8 kg and 3.4 kg.