From November 2005 Paper 2 [calculator okay]

(a) Given that $\dfrac{x^2}{(1+x)(1+x^2)} = \dfrac{a}{(1+x)} + \dfrac{bx+c}{(1+x^2)}$,

calculate the value of $a, b,$ & c

(b) Hence, find $I = \int \dfrac{x^2}{(1+x)(1+x^2)}\,dx$

(c) If $I = \dfrac{\pi}{4}$ when $x = 1$, calculate the value of the constant of integration in the form $p + q\ln r$ where $p, q, r \in R$

(a) $$\frac{x^2}{(1+x)(1+x^2)} = \frac{a}{1+x} + \frac{bx+c}{(1+x^2)}$$

$$x^2 = a(1+x^2) + (bx+c)(1+x)$$

$$x^2 = x^2(a+b) + x(b+c) + a+c$$

$a+b=1$ $\qquad a = \frac{1}{2}$

$b+c=0$ $\qquad b = \frac{1}{2}$

$a+c=0$ $\qquad c = -\frac{1}{2}$

(b) $$I = \frac{1}{2}\int \left[\frac{1}{1+x} + \frac{x-1}{(1+x^2)}\right] dx$$

$$= \frac{1}{2}\int \frac{1}{1+x}\,dx + \frac{1}{2}\left[\int \frac{x}{1+x^2}\,dx - \int \frac{1}{1+x^2}\,dx\right]$$

$$= \frac{1}{2}\ln|1+x| + \frac{1}{4}\ln(1+x^2) - \frac{1}{2}\arctan x + C$$

$$\frac{\pi}{4} = \frac{1}{2}\ln 2 + \frac{1}{4}\ln 2 - \frac{\pi}{8} + C$$

$$\frac{3\pi}{8} = \frac{3}{4}\ln 2 + C$$

$$\frac{3\pi}{8} - \frac{3}{4}\ln 2 = C$$

$p = \frac{3\pi}{8}$ $\quad q = -\frac{3}{4}$ $\quad r = 2$

May 2005 TZ1 Paper 2 [calculator okay]

(a) The function g is defined by $g(x) = \frac{e^x}{\sqrt{x}}$ for $0 < x \leq 3$

(i) Sketch the graph of g

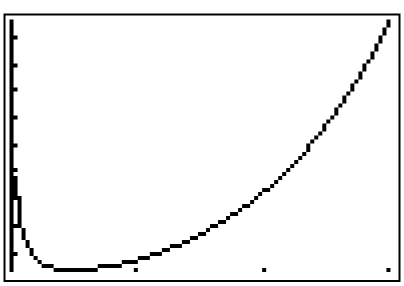
(ii) Find $g'(x)$

(iii) Write down an expression representing the gradient of the normal to the curve at any point

(b) Let P be the point (x, y), and Q the point $(1, 0)$

(i) Find the gradient of (P, Q) in terms of x

(ii) Given that the line (PQ) is a normal to the graph of g at the point P, find the minimum distance from the point Q to the graph of g



$y = \frac{e^x}{\sqrt{x}}$ on $(0, 3]$

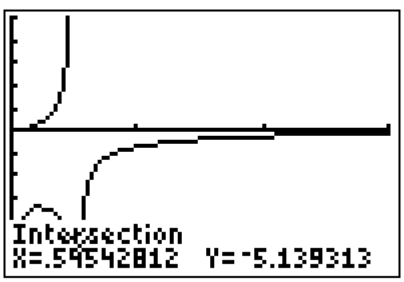
$$g'(x) = \frac{\sqrt{x}\,e^{x} - \frac{e^{x}}{2\sqrt{x}}}{x} = \frac{(2x-1)e^{x}}{2x\sqrt{x}}$$

$$\text{GRADIENT TO NORMAL} = -\frac{1}{g'(x)} = -\frac{2x\sqrt{x}}{(2x-1)e^{x}}$$

SLOPE OF LINE PQ

$$\frac{y-0}{x-1} = \frac{e^{x}}{\sqrt{x}}\left(\frac{1}{x-1}\right)$$

$$= \frac{e^{x}}{\sqrt{x}(x-1)}$$



$$\frac{e^{x}}{\sqrt{x}(x-1)} = \frac{-2x\sqrt{x}}{(2x-1)e^{x}}$$

at $x \approx .5454$

$$\text{DISTANCE} = \sqrt{(1-.5454)^2 + \left(\frac{e^{.5454}}{\sqrt{.5454}}\right)^2}$$

$$\approx 2.38$$

May 2006 Paper 2 [calculator]

Particle A moves in a straight line, starting from O_A, such that its velocity in metres per second for $0 \leq t \leq 9$ is given by

$$v_A = -\frac{1}{2}t^2 + 3t + \frac{3}{2}$$

Particle B moves in a straight line starting from O_B, such that its velocity in metres per second for $0 \leq t \leq 9$ is given by

$$v_B = e^{0.2t}$$

(a) Find the maximum value of v_A, justifying that it is a maximum [not with a graphing calculator]

(b) Find the acceleration of B when $t = 4$

The displacements of A and B from O_A and O_B respectively, at time t are s_A metres and s_B metres. When t = 0, $s_A = 0$ and $s_B = 5$

(c) Find an expression for s_A and s_B, giving your answers in terms of t

(d) Sketch the curves of s_A and s_B on the same diagram and find the values of t at which $s_A = s_B$