IBHL Review Two

Let $z = \cos\theta + i\sin\theta$, for $\frac{-\pi}{4} < \theta < \frac{\pi}{4}$

(a)(i) Find z^3 using the binomial theorem

(ii) Use deMoivre's Theorem to show that

$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$ and
$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

(b) Hence, prove that $\frac{\sin 3\theta - \sin\theta}{\cos 3\theta + \cos\theta} = \tan\theta$

(c) Given that $\sin\theta = \frac{1}{3}$, find the exact value of

$\tan 3\theta$

Consider the system of equations

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -42 \end{pmatrix} \text{where } T = \begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & r \\ 3r & 0 & s \end{pmatrix}$$

(a) Find the solution of the system when $r = 0,\ s = 3$

$$\begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -42 \end{pmatrix}$$

$-x + 3y = 4$

$2y = -2$

$3z = -42$

$x = -7$

$y = -1$

$z = -14$

(b) The solution of the system is not unique

(i) Show that $s = \frac{9}{2}r^2$

let $\det T = 0$

$\det T = -2s + 9r^2$

$0 = -2s + 9r^2$

$\frac{9}{2}r^2 = s$

(ii) When r = 2 and s = 18, show that the system can be solved, and find the general solution

$$\begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & 2 \\ 6 & 0 & 18 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -42 \end{pmatrix}$$

$$-x + 3y = 4$$
$$2y + 2z = -2$$
$$6x + 18z = -42$$

$$\begin{array}{rrrl} & 18y & +18z & = -18 \\ 6x & & +18z & = -42 \\ \hline -6x & +18y & & = 24 \end{array}$$

$$\begin{array}{rrl} -6x & +18y & = 24 \\ -6x & +18y & = 24 \\ \hline & 0 & = 0 \end{array}$$

let $z = t$

$$2y + 2z = -2$$
$$z = -1 - y$$
$$y = -1 - t$$
$$x = -7 - 3t$$

IF $y = t$; $z = -1 - t$; $x = 3t - 4$

IF $x = t$; $y = \frac{4+t}{3}$; $z = \frac{-7-t}{3}$

(c) Use Mathematical Induction to prove that, when r = 0

$$T^n = \begin{pmatrix} (-1)^n & 2^n - (-1)^n & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & s^n \end{pmatrix} \text{ for n } \in \mathbb{Z}^+$$

$$S(1) = T^1 = \begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & s \end{pmatrix}$$

$$= \begin{pmatrix} (-1)^1 & 2^1 - (-1)^1 & 0 \\ 0 & 2^1 & 0 \\ 0 & 0 & s \end{pmatrix}$$

So $S(1)$ is TRUE

ASSUME $S(K)$ is TRUE i.e, ...

WE MUST SHOW THAT $S(K+1) = T^{K+1}$

$$= \begin{pmatrix} (-1)^{K+1} & 2^{K+1} - (-1)^{K+1} & 0 \\ 0 & 2^{K+1} & 0 \\ 0 & 0 & s^{K+1} \end{pmatrix}$$

$$T^{K+1} = T^K \cdot T^1$$

$$= \begin{pmatrix} (-1)^K & 2^K - (-1)^K & 0 \\ 0 & 2^K & 0 \\ 0 & 0 & s^K \end{pmatrix} \begin{pmatrix} -1 & 2^1 - (-1)^1 & 0 \\ 0 & 2^1 & 0 \\ 0 & 0 & s \end{pmatrix}$$

$$= \begin{pmatrix} (-1)^{K+1} & 2(2^K) - (-1)^{K+1} & 0 \\ 0 & 2^{K+1} & 0 \\ 0 & 0 & s^{K+1} \end{pmatrix}$$

blah blah Hence thus Therefore...