More IBHL Review Stuff

A big non-calculator problem:

If z is a non-zero complex number, we define

$L(z) = \ln|z| + i\arg(z)$ for $0 \le \arg(z) < 2\pi$

(a) Show that when z is a positive real number, $L(z) = \ln z$

(b) Use the equation to calculate

$L(-1),\ L(1-i),\ L(-1+i)$

(c) Hence, show that the property $L(z_1 z_2) = L(z_1) + L(z_2)$ does NOT hold for all values of z_1 *and* z_2

Consider the matrix $A = \begin{pmatrix} e^x & e^{-x} \\ 2+e^x & 1 \end{pmatrix}$ where $x \in R$

Find the value of x for which A is singular

Find the vector equation of the line of intersection of the three planes represented by the following system of equations

This is a G.D.C. problem

$$\begin{aligned} 2x &\quad -7y &\quad +5z &= 1 \\ 6x &\quad +3y &\quad -z &= -1 \\ -14x &\quad -23y &\quad +13z &= 5 \end{aligned}$$

Find the gradient of the tangent line to the curve $x^3 y^2 = \cos(\pi y)$ at the point $(-1, 1)$

Consider the set of numbers a, $2a$, $3a$, …, na where a and n are positive integers.

(a) Show that the expression for the mean of this set is $\frac{a(n+1)}{2}$

(b)Let $a = 4$. Find the minimum value of n for which the sum of these numbers exceeds its mean by more than 100