IBHL Maths Probability Day Two

Dependent Events

Two or more events are dependent if they are not independent. [Did a lawyer write this?]

Dependent events are events for which the occurrence of one of the events does affect the occurrence of the other event.

If A and B are dependent events then

$$P(A \text{ then } B) = P(A) \times P(B \text{ given that } A \text{ has occurred})$$

Example:

A hat contains tickets with the numbers 1, 2, 3, …, 19, 20 on them. If three tickets are taken from the hat, without replacement, what is the probability that all three tickets have prime numbers on them?

Our sample space:

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]

There are 8 prime numbers in our sample space.

$$P(3\ primes) = \left(\frac{8}{20}\right)\left(\frac{7}{19}\right)\left(\frac{6}{18}\right)$$

$$= \frac{336}{6840} \text{ or } \frac{14}{285}$$

Another example from our textbook:

A bin contains 12 identically shaped chocolates of which 8 are strawberry creams. If 3 chocolates are selected simultaneously from the bin, determine the probability that:

a they are all strawberry creams

b none of them are strawberry creams.

TOTAL OF 12

(a) $P(SSS) = \left(\frac{8}{12}\right)\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)$

$= \frac{336}{1320}$

$P(NNN) = \left(\frac{4}{12}\right)\left(\frac{3}{11}\right)\left(\frac{2}{10}\right)$

How to use a tree diagram with compound events:

Example 10

Carl is not having much luck lately. His car will only start 80% of the time and his motorbike will only start 60% of the time.

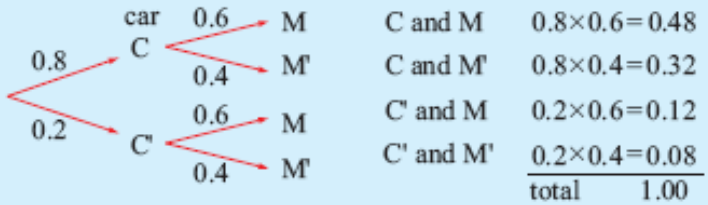
a Draw a tree diagram to illustrate this situation.

b Use the tree diagram to determine the chance that:

i both will start **ii** Carl has no choice but to use his car.

a C = car starts

M = motorbike starts



outcome	probability
C and M	$0.8 \times 0.6 = 0.48$
C and M'	$0.8 \times 0.4 = 0.32$
C' and M	$0.2 \times 0.6 = 0.12$
C' and M'	$0.2 \times 0.4 = 0.08$
total	1.00

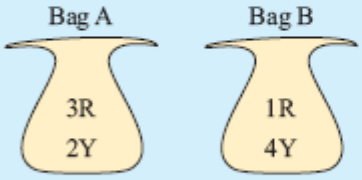
b **i** P(both start)
$= P(C \text{ and } M)$
$= 0.8 \times 0.6$
$= 0.48$

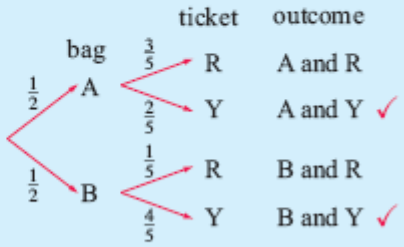
ii P(car starts but motorbike does not)
$= P(C \text{ and } M')$
$= 0.8 \times 0.4$
$= 0.32$

If there is more than one outcome in an event then we need to **add** the probabilities of these outcomes.

Example 11

Bag A contains 3 red and 2 yellow tickets. Bag B contains 1 red and 4 yellow tickets. A bag is randomly selected by tossing a coin, and one ticket is removed from it. Determine the probability that it is yellow.





$P(\text{yellow}) = P(\text{A and Y}) + P(\text{B and Y})$

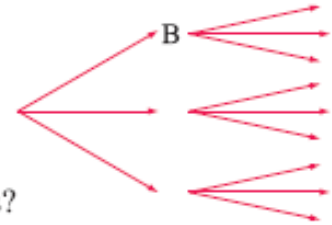
$= \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{4}{5}$ {branches marked with a ✓}

$= \frac{3}{5}$

Let's give some a try!

Suppose this spinner is spun twice.

a Copy and complete the branches on the tree diagram shown.

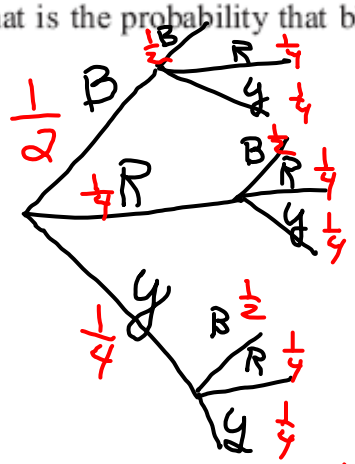


b What is the probability that black appears on both spins?

c What is the probability that yellow appears on both spins?

d What is the probability that different colours appear on the two spins?

e What is the probability that black appears on either spin?



$P(B \text{ and } B) = P(B)\,P(B)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{4}$

$P(y \text{ and } y) = P(y)\,P(y)$

$= \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$

$= \frac{1}{16}$

$P(\text{DIFF COLOURS})$

$= P(BR) + P(By) + P(RB) + P(Ry) + P(yB) + P(yR)$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16}$

$= \frac{5}{8}$

$P(\text{B on either})$

$= P(BB) + P(BR) + P(BG) + P(RB) + P(GB)$

$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= \frac{3}{4}$

Sampling with and without replacement

Sampling is the process of selecting an object from a large group of objects and noting its features. The object is then either put back [sampling with replacement] or put aside [sampling without replacement]

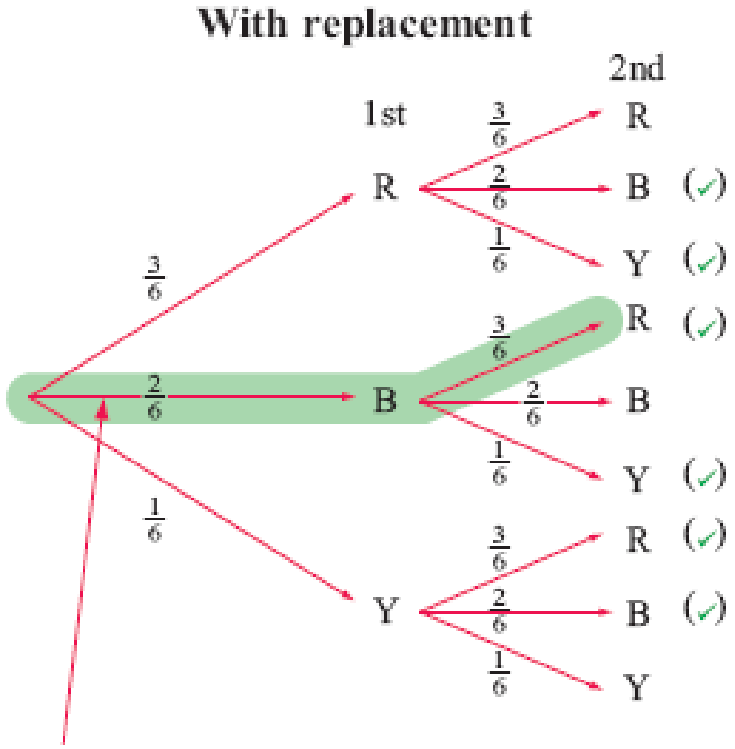
In real-life this is commonly used for quality control – namely, to see if a manufactured object is without defect.

Consider a box containing 3 red, 2 blue, and 1 yellow marble. Suppose you want to sample two marbles.

(a) By replacement of the first before the second is drawn

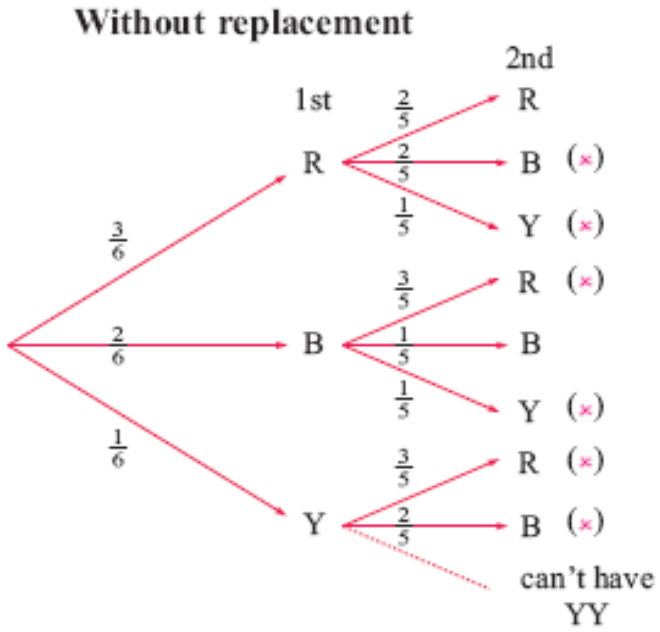
(b) By not replacing the first before the second is drawn

Find P(two reds)



$P(RR) = \left(\frac{3}{6}\right)\left(\frac{3}{6}\right)$

The green highlighted line shows blue, then red as the drawn marbles



$P(RR) = \left(\frac{3}{6}\right)\left(\frac{2}{5}\right)$

Another example to ponder:

5 tickets numbered 1, 2, 3, 4, and 5, are placed in a hat. Two tickets are taken from the bag without replacement. Determine the following probabilities:

(a) P(odd,odd) $= \left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{3}{10}$

(b) P (even, even) $= \left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{10}$

(c) P(one odd and the other even)

$= 1 - P(\text{Odd Odd}) - P(\text{Even Even})$

$= 1 - \frac{4}{10}$

$= \frac{6}{10}$

Skippy has a bag of sweets [candies] which are all identical in shape. The bag contains 6 orange drops and 4 lemon drops. Skippy selects one sweet at random, eats it, and then takes another, also at random. Determine the probability that

(a) P(orange, orange) $= \left(\frac{6}{10}\right)\left(\frac{5}{9}\right)$

(b) P (lemon, lemon) $= \left(\frac{4}{10}\right)\left(\frac{3}{9}\right)$

(c) P(orange then lemon) $= \left(\frac{6}{10}\right)\left(\frac{4}{9}\right)$

(d) P(lemon then orange) $= \left(\frac{4}{10}\right)\left(\frac{6}{9}\right)$

(e) P(Skippy will be fat) EITHER 0% OR 100%

(f) Add the probabilities from (a) to (d)

$$= \frac{30+24+24+12}{90}$$

$$= 1$$