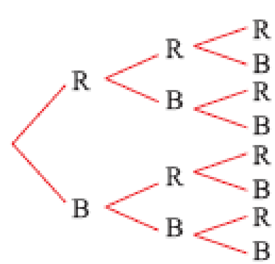
Binomial Probabilities

If *E* is an event with probability *p* of occurring and *E'* has probability *q* where clearly *p+q=1* then the probability generator for the various outcomes over n independent trials is $(p+q)^n$

Consider rolling a die 3 times. The die has two red faces and the other four faces are black. If R represents "the result is red" and B represents "the result is black", then what are the possible outcomes.



Event	*Outcome*	*Probabilities*	*Total Probability*
all red	RRR	$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$	$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$
2 red and 1 black	BRR RBR RRB	$\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$ $\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$ $\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$	$3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) = \frac{6}{27}$
1 red and 2 black	RBB BRB BBR	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$ $\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$ $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$	$3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 = \frac{12}{27}$
all black	BBB	$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

Notice that $\left(\frac{1}{3}\right)^3 + 3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3$ is the binomial expansion for $\left(\frac{1}{3} + \frac{2}{3}\right)^3$.

Holy smokes! Pascal's Triangle strikes again!

Do you remember the big “egg farm” scandal of last summer?

If E is the event of a randomly chosen chicken egg being “faulty” and P(E)=0.03 $= p$ and P(E’)=0.97$= q$, then find the probabilities if four eggs are taken as samples.

$$P(E \text{ occurs } x \text{ times and } E' \text{ occurs } n-x \text{ times}) = \binom{n}{x} p^x q^{n-x}$$

So for four samples,

[This is sometimes called the probability generator]

$$P(4E)+P(3E+1E')+P(2E+2E')+P(1E+3E')+P(4E')$$

$$= (.03)^4 + 4(.03)^3(.97) + 6(.03)^2(.97)^2 + 4(.03)(.97)^3 + (.97)^4$$

$$= 1$$

An archer has a 90% chance of hitting the target with each arrow and if 5 arrows are used, determine the probability generator and hence the chance of hitting the target

(a) Twice only $P(H=2) = 10(.9)^2(.1)^3 = .0081$

(b) At most 3 times $1 - P(5H) - P(4H)$

$= 1 - (.9)^5 - 5(.9)^4(.1)$

$\approx .0815$

Let H be the event of "hitting the target"

$P(H) = .9$

$P(H') = .1$

Probability generator:

$P(5H) + P(4H) + P(3H) + P(2H) + P(1H) + P(0H)$

$= (.9)^5 + \binom{5}{1}(.9)^4(.1) + \binom{5}{2}(.9)^3(.1)^2$

$+ \binom{5}{3}(.9)^2(.1)^3 + \binom{5}{4}(.9)(.1)^4$

$+ (.1)^5$

If a random experiment has only two results [success or failure], and n number of independent trials are carried out, the distribution of successes is called the binomial distribution.

$$P(x\, successes\ in\ n\ trials) = P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

For $x = 0, 1, 2, 3, \ldots, n$

This can also be notated as $X \sim B(n, p)$

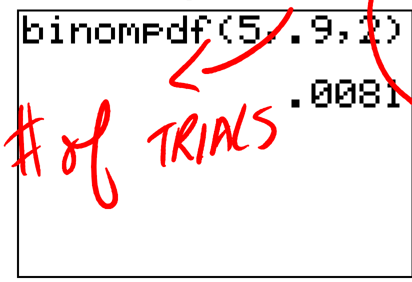
The B stands for "binomial", the n is the number of trials, and the p is the probability of success

As luck would have it, our G.D.C. will find these for us. The binomial probability distribution function can be found under DISTR on our

G.D.C.   Let's look at the archer problem again, this time using our G.D.C.

$P(2H)$ can be thought of as

Binompdf(5, 0.9, 2) # of successes we want

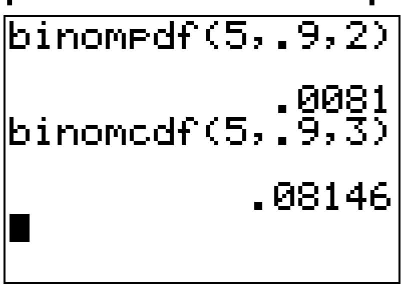


# of TRIALS

P(H)

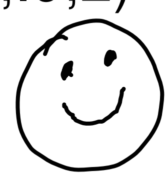
P(H ≤ 3)

To find $P(X \le 3)$ we will need to choose binomcdf which stands for binomial cumulative distribution function. This adds up the probabilities up to and including P(3H)



We can check this by adding up binompdf(5, .9, 0)+binompdf(5, .9, 1)+binompdf(5,.9,2) +binompdf(5,.9,3)

Try it!



MULTIPLE GUESS

In a multiple choice test there are 10 questions and each question has 5 choices, one of which is correct. If 70% is the pass mark and Skippy [who never takes notes, doesn't study, and doesn't do homework] guesses at each answer, then determine the probability that Skippy will pass.

Note: $P(X \geq 7) = 1 - P(X \leq 6)$

TI: 1- binomcdf (10, .2, 6)

P(SKIPPY GUESSES AND PASSES)

$\approx .00864$

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binomcdf(10,.2,6
)
      .9991356416
Ans→A
      .9991356416
1-A
      8.643584E-4
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[Hence, the moral of this story is that maybe Skippy should be more studious]

A computer shop orders its laptops from a supplier that has a 10% rate of defective items. The shop usually takes a sample of 10 laptops and checks them for defects. If they find 2 items defective, they send the shipment back. What is the probability that their sample will need to be sent back?

$P(\text{sent BACK}) \approx 26.39\%$

$1 - P(1) - P(0)$

José likes to eat pizza. When buying a pizza he can choose one of three companies *A*, *B* and *C*. 60% of the time he will chose company *A*, 30% he will chose *B* and 10% he will chose *C*. Each company has a 30-minute delivery policy, i.e. if the pizza arrives after 30 minutes it will be free. For company *A*, 6% of the time the pizza is free and for *B* and *C* it is 5% and 3% respectively.

a. Draw a probability tree diagram clearly labelling the probability of each outcome.

Calculate the probability that

b. the next pizza from company *A* will be free, $P(A \text{ Free}) = .6(.06)$

$P(A \cap \text{Free})$

c. the next pizza will be free.

$= P(A \cap \text{Free}) + P(B \cap \text{Free}) + P(C \cap \text{Free}) = .054$

Given that the pizza is free, find, the probability that it came from company *B*.

$P(A) = 60\%$; $P(B) = 30$; $P(C) = 10\%$

$P(A \text{ FREE}) = 6\%$; $P(B \text{ Free}) = 5\%$; $P(C \text{ Free}) = 3\%$