Laws of Probability

For 2 events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

In words,

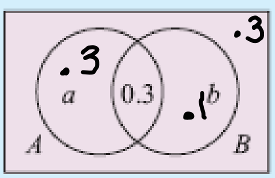
P(Either A or B)=P(A)+P(A)-P(Both A and B)

This is easy to see on a Venn Diagram

If $P(A) = 0.6$, $P(A \cup B) = 0.7$ and $P(A \cap B) = 0.3$, find $P(B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\therefore \quad 0.7 = 0.6 + P(B) - 0.3$$
$$\therefore \quad P(B) = 0.4$$

or



Using a Venn diagram with the probabilities on it,

$a + 0.3 = 0.6$ and $a + b + 0.3 = 0.7$

$\therefore \quad a = 0.3$ $\qquad \therefore \quad a + b = 0.4$

$\therefore \quad 0.3 + b = 0.4$

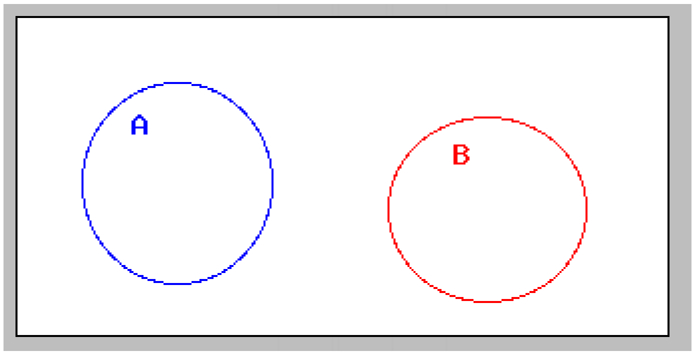
$\therefore \quad P(B) = 0.3 + b = 0.4$ $\qquad \therefore \quad b = 0.1$

If A and B are mutually exclusive events, then

$$P(A \cap B) = 0$$

Hence, $P(A \cup B) = P(A) + P(B)$

If A and B are mutually exclusive, then what does the Venn Diagram look like?



Sometimes these are called disjoint sets.

Example:

A box of chocolates contains 6 with hard centres (H) and 12 with soft centres (S).

Are the events H and S mutually exclusive? yes

Find: P(H), P(S), $P(H \cap S)$, $P(H \cup S)$

$P(H) = \frac{6}{18}$; $P(S) = \frac{12}{18}$

$P(H \cap S) = 0$

$P(H \cup S) = 1$

Conditional Probability

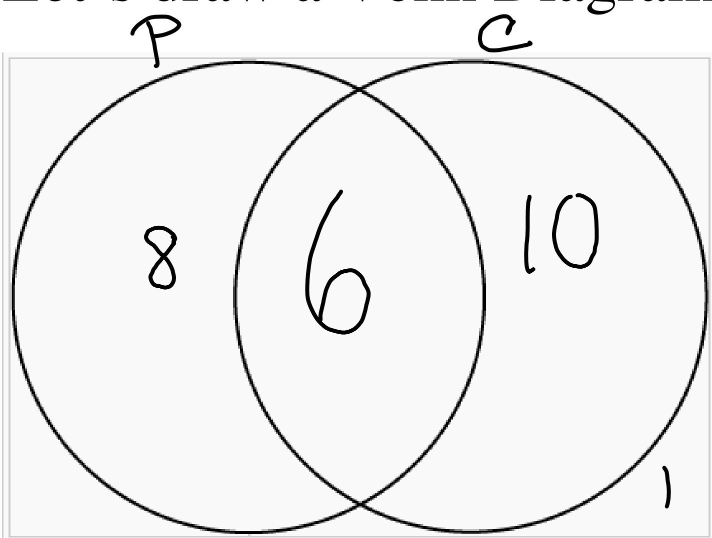
$A \mid B$ is read as "A given B" and represents

"A occurs knowing that B occurs"

In a class of 25 students, 14 like pizza and 16 like coffee. One student likes neither and 6 students like both.

Find the probability that a random student likes pizza, $P(pizza)$, and find the probability that a student likes pizza given that they like coffee?

Let's draw a Venn Diagram first.



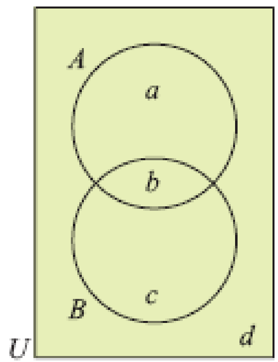
$P(pizza) = \frac{14}{25}$

$P(pizza \mid coffee) = \frac{6}{16}$

If A and B are events then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Proof:



$$\begin{aligned}
& P(A \mid B) \\
&= \frac{b}{b+c} \qquad \{\text{Venn diagram}\} \\
&= \frac{b/(a+\cancel{b}+c+d)}{(b+c)/(a+\cancel{b}+c+d)} \\
&= \frac{P(A \cap B)}{P(B)}
\end{aligned}$$

It follows that $P(A \cap B) = P(A \mid B)P(B)$

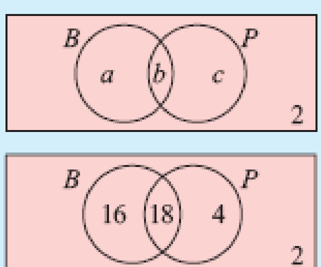
In a class of 40 students, 34 like bananas, 22 like pineapples, and 2 dislike both fruits. If a student is randomly selected, find the probability that the student:

a likes both fruits

b likes at least one fruit

c likes bananas given that he or she likes pineapples

d dislikes pineapples given that he or she likes bananas.



P(B∩P) P(P∪B)

B represents students who like bananas.
P represents students who like pineapples.

We are given that
$$a+b=34$$
$$b+c=22$$
$$a+b+c=38$$

$\therefore \; c = 38-34 = 4$ and so $b = 18$ and $a = 16$

a P(likes both) $= \frac{18}{40} = \frac{9}{20}$

b P(likes at least one) $= \frac{38}{40} = \frac{19}{20}$

c $P(B \mid P) = \frac{18}{22} = \frac{9}{11}$

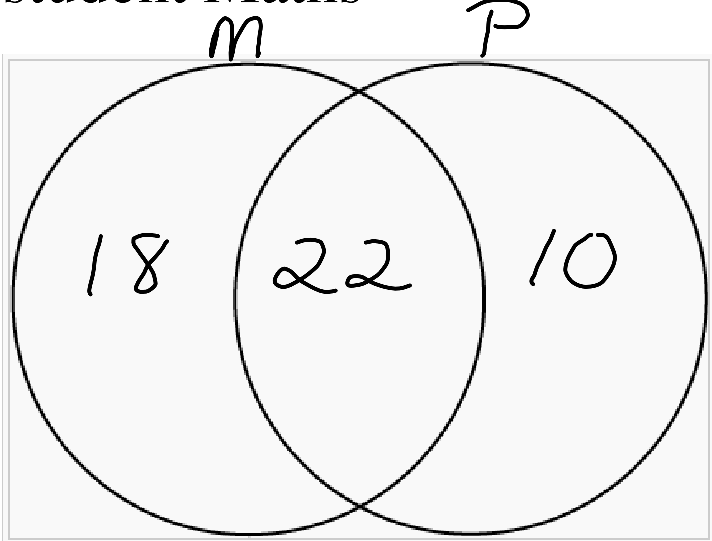
d $P(P' \mid B) = \frac{16}{34} = \frac{8}{17}$

In a group of 50 students, 40 study Maths, 32 study Physics, and each student studies at least one of these subjects.

Draw a Venn Diagram to illustrate this and then find the following probabilities:

(a) A student studies Maths but not Physics

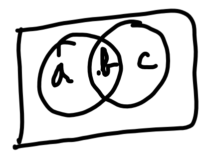
(b) A student studies Physics given that they student Maths



$a + b + c = 50$

$a + b = 40$

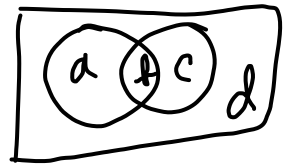
$b + c = 32$

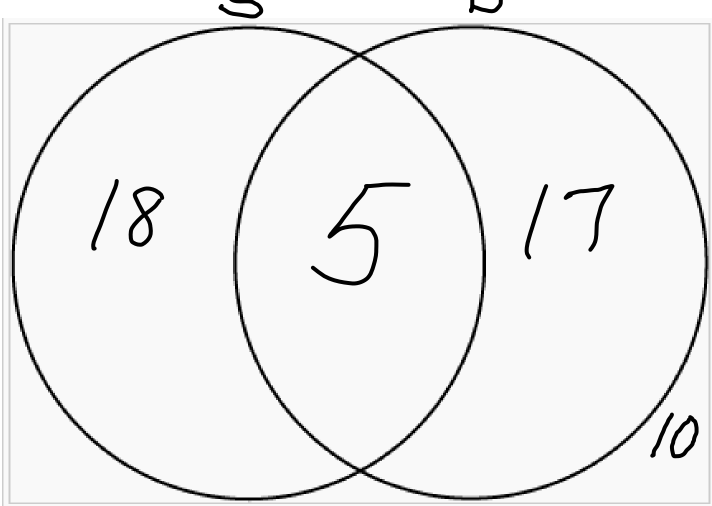


$P(\text{MATHS BUT NOT PHYSICS}) = \frac{18}{50}$

$P(P \mid M) = \frac{22}{40}$

50 students go “bushwalking”. 23 get a sunburn, 22 get bitten by ants, and 5 unlucky students are both bitten and sunburned. Draw a Venn Diagram and then find the following probabilities:





(a) P(not bitten) $= \frac{28}{50}$

(b) P(either bitten or sunburned) $= \frac{40}{50}$

$\sim P(B \cup S)$

(c) P(was bitten, given that they were sunburned) $= \frac{5}{23}$

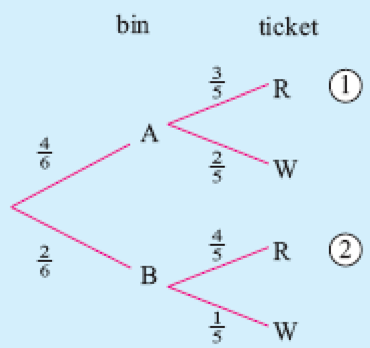
(d) P(was sunburned, given that not bitten)

$= \frac{18}{28}$

You can also use tree diagrams to do these kind of problems.

Bin A contains 3 red and 2 white tickets. Bin B contains 4 red and 1 white ticket. A die with 4 faces marked A and two faces marked B is rolled and used to select bin A or B. A ticket is then selected from this bin. Determine the probability that:

a the ticket is red **b** the ticket was chosen from B given it is red.



a $\text{P}(R)$

$= \frac{4}{6} \times \frac{3}{5} + \frac{2}{6} \times \frac{4}{5}$ {path 1 + path 2}

$= \frac{2}{3}$

b $\text{P}(B \mid R) = \dfrac{\text{P}(B \cap R)}{\text{P}(R)}$

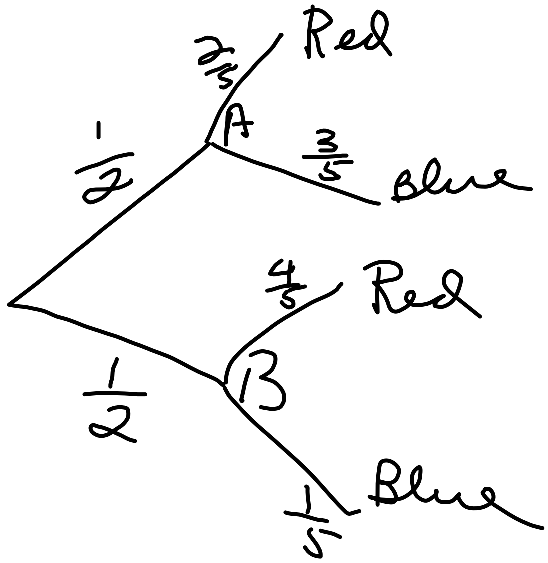
$= \dfrac{\frac{2}{6} \times \frac{4}{5}}{\frac{2}{3}}$ ← path 2

$= \frac{2}{5}$

Bin A contains 2 red Skittles and 3 blue Skittles, and Bin B contains 4 red Skittles and 1 blue Skittle. Skippy selects a bin by tossing a coin and then takes a Skittle from that bin.

(a) Determine the probability that the Skittle is red.

(b) Given that the Skittle is red, what is the probability that it came from Bin B?



$$P(\text{Red}) = \left(\tfrac{1}{2}\right)\left(\tfrac{2}{5}\right) + \left(\tfrac{1}{2}\right)\left(\tfrac{4}{5}\right) = \frac{3}{5}$$

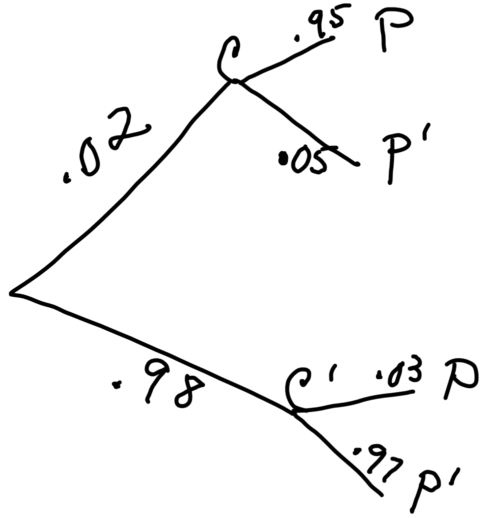
$$P(B|R) = \frac{P(B \cap R)}{P(R)} = \frac{\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)}{\frac{3}{5}} = \frac{2}{3}$$

The probability that a randomly selected person has cancer is 0.02. The probability that he/she reacts positively to a test which detects cancer is 0.95 if he/she has cancer and 0.03 if he/she does not have cancer. Determine the following probabilities:

(a) P(reacts positively) $= (.02)(.95) + (.98)(.03) = .0484$

(b) P(has cancer given that he/she reacts positively)

$$= \frac{(.02)(.95)}{.0484} \approx .3926$$



.02
C
.95
P
.05
P'
.98
C'
.03
P
.97
P'