

Non-calculator problems

Consider the following matrices:

$$A = \begin{pmatrix} 2 & 5 & 1 \\ 0 & -3 & 2 \\ 7 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} m & -2 \\ 3m & -1 \\ 2 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} x-1 & 5 & y \\ 0 & -x & y+1 \\ 2x+y & x-3y & 2y-x \end{pmatrix}$$

- (a) Find $A + C$
- (b) Find AB
- (c) Find BA
- (d) Solve for x and y if $A = C$
- (e) Find $B + C$

(f) Solve for m if $3B + 2 \begin{pmatrix} -1 & m^2 \\ -5 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 7 & 12 \\ 17 & 1 \\ 2m+2 & 7 \end{pmatrix}$

Solve for x and y such that $AB = BA$ if $A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} x & 2 \\ y & 3 \end{pmatrix}$

Find the value(s) of m so that the matrix $\begin{pmatrix} 1 & m & 1 \\ 3 & 1-m & 2 \\ m & -3 & m-1 \end{pmatrix}$

The matrices A, B are such that $\det A = \det B$ where $A = \begin{pmatrix} 1 & -3 \\ x+1 & 5 \end{pmatrix}$ and

$$B = \begin{pmatrix} 1 & x & -2 \\ -2 & 1 & 0 \\ 0 & -1 & x \end{pmatrix}.$$

Find the values of x

Calculator

Given that $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$, find X if $BX = A - AB$

2006 TZ0 Paper 2 Question 5

Consider the system of equations $T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ -42 \end{pmatrix}$ where $T = \begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & r \\ 3r & 0 & s \end{pmatrix}$

- (a) Find the solution of the system when $r = 0$ and $s = 3$
- (b) The solution of the system is not unique
 - (i) Show that $s = \frac{9}{2} r^2$
 - (ii) When $r = 2$ and $s = 18$, show that the system can be solved, and find the general solution [To make it easy to check, let $z = t$ when writing your general solution. Hint: You'll need the Gauss-Jordan elimination method]
- (c) Use mathematical induction [our favorite] to prove that when $r = 0$

$$T^n = \begin{pmatrix} (-1)^n & 2^n - (-1)^n & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & s^n \end{pmatrix} \text{ where } n \in \mathbb{Z}^+$$