**Mathematical Induction for IBHL Maths**
**Day Two**

**Let's look at some problems in our textbook**
**Please see Chapter 10, page 228**

**1B[Warm-up]**
**Using PMI show that**

$$1^2+3^2+5^2+7^2+\ldots+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$$

**for** $n\in Z^+,\ n\geq 1$

**Let** $S(n)$ **be the statement that for** $n\in Z^+$**,**

$$1^2+3^2+5^2+7^2+\ldots+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$$

$S(1)$

**Assume that for** $n=k,\ S(k)$ **is true. In other words, for** $k\in Z^+$**,**

$$1^2 + 3^2 + 5^2 + 7^2 + \ldots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

**We must show that**

Show, using PMI, that for all non-negative integers

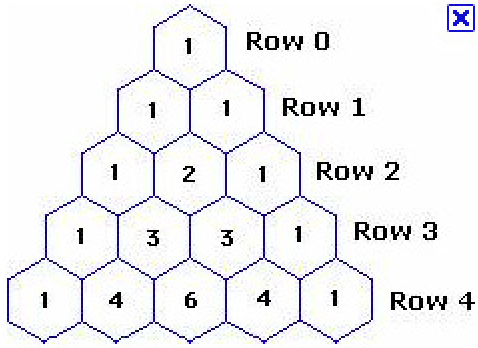
$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n}=2^n$$

[Hey, it's Pascal's Triangle!]

Notice that this one is slightly different because were are not just considering $n \in Z^+$

Instead of $S(1)$ as our first step, we need to show that $S(0)$ is true as our first step.

Keep thinking about Pascal's Triangle.




Notice that $\binom{3}{1}+\binom{3}{2}=\binom{4}{2}$

In general, $\binom{n}{r-1}+\binom{n}{r}=\binom{n+1}{r}$

We are going to need this!

What do you get when you add:

$$\binom{k}{0}+\binom{k}{1}+\binom{k}{2}+\dots+\binom{k}{k-1}+\binom{k}{k}=2^k$$

$$\binom{k}{0}+\binom{k}{1}+\binom{k}{2}+\dots+\binom{k}{k-1}+\binom{k}{k}=2^k$$

$$\binom{k}{0}+\binom{k+1}{1}+\binom{k+1}{2}+\dots+\binom{k+1}{k}+\binom{k}{k}=2\cdot 2^k$$

But, since $\binom{k}{0}=\binom{k+1}{0}=\binom{k}{k}=\binom{k+1}{k+1}=1$, then

we can rewrite this as:

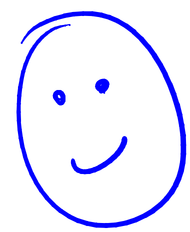
Another one to ponder:
Show that in an arithmetic sequence where $a_n = a_{n-1} + d$, the $n^{th}$ term can be given by the formula $a_n = a_1 + (n-1)d$

Now do #2 and 4 on page 228 on your own

#2

Using PMI show that for $n \in Z^+$, $3^{2n+2} - 8n - 9$ is divisible by 64.

Let $S(n)$ be the statement that for $n \in Z^+$, $3^{2n+2} - 8n - 9$ is divisible by 64.



Is $S(1)$ true?

$S(1) = 3^{2+2} - 8 - 9$

$= 64$ WHICH IS DIVISIBLE BY 64

Assume that $S(k)$ is true for $k = n$, $3^{2k+2} - 8k - 9$ is divisible by 64. We must show that $S(k+1)$ is divisible by 64 [using the fact that $S(k)$ is divisible by 64]

Here goes:

$$S(k+1) = 3^{2(k+1)+2} - 8(k+1) - 9$$

$$= 3^{2k+2+2} - 8k - 8 - 9$$

$$= 3^{2k+2} \cdot 3^2 - 8k - 17$$

$$= 3^{2k+2} \cdot 3^2 - 8k - 17 + 64k - 64k + 72 - 72$$

Let $3^{2k+2} - 8k - 9 = 64m$

$$= 3^{2k+2} \cdot 3^2 - 72k - 81 - 8 + 64k + 72$$

$$= 9\underbrace{\left(3^{2k+2} - 8k - 9\right)} + 64k + 64$$

$$= 9(64m) + 64k + 64$$

Hence $S(k+1)$ is a multiple of 64

Hence, since $S(k)$ implies that $S(k+1)$ is true for $k \in Z^+$, then by the principle of mathematical induction $S(n)$ is true for all $n \in Z^+$

#4

Show that for $n = 0, 1, 2, 3, \ldots$ $5^n + 3$ is divisible by 4

Let $S(n)$ be the statement that for $n = 0, 1, 2, 3, \ldots$ $5^n + 3$ is divisible by 4

Is $S(0)$ true?

$S(0) = 5^0 + 3$

$= 4$ IS DIVISIBLE by 4

Assume that $S(k)$ is true for $k = n$, namely that for $k = 0, 1, 2, 3, \ldots$ $5^k + 3$ is divisible by 4

We must show that $S(k+1)$ is divisible by 4

Here goes:

$$
\begin{aligned}
S(k+1) &= 5^{k+1} + 3 \\
&= 5^k \cdot 5 + 3 + 12 - 12 \\
&= 5^k \cdot 5 + 15 - 12 \\
&= 5(5^k + 3) - 12 \\
&= 5(4m) - 12
\end{aligned}
$$

let $5^k + 3 = 4m$

WHICH IS DIVISIBLE by 4

Hence, since $S(k)$ implies that $S(k+1)$ is true for $k \in 0, 1, 2, \ldots$, then by the principle of mathematical induction $S(n)$ is true for all $n = 0, 1, 2, \ldots$

GRADUATION = 5/22
SUNDAY ≈ 2 PM

IBHL MATHS

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