Mathematical Induction for IBHL Maths
Day Two

Let's look at some problems in our textbook Please see Chapter 10, page 228

1B[Warm-up]
Using PMI show that

$$1^2+3^2+5^2+7^2+\ldots+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$$

for $n\in Z^+,\ n\geq 1$

Let $S(n)$ **be the statement that for** $n\in Z^+$**,**

$$1^2+3^2+5^2+7^2+\ldots+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$$

$S(1)\ = 1\ = \frac{1(3)(1)}{3}$ yes!

S(1) is TRUE

Assume that for $n=k,\ S(k)$ **is true. In other words, for** $k\in Z^+$**,**

$$1^2+3^2+5^2+7^2+\ldots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$$

We must show that

$$S(K+1)=\frac{(K+1)(2(K+1)-1)(2(K+1)+1)}{3}$$

$$S(K+1)=$$

$$\underbrace{1^2+3^2+5^2+7^2+\ldots+(2K-1)^2}+(2(K+1)-1)^2=\frac{(K+1)(2(K+1)-1)(2(K+1)+1)}{3}$$

$$\frac{(K+1)(2K+3)(2K+1)}{3}$$

$$S(K)+(2(K+1)-1)^2=$$

$$\frac{K(2K+1)(2K-1)}{3}+(2(K+1)-1)^2=$$

$$\frac{K(2K+1)(2K-1)}{3}+\frac{3\overset{(2K+1)}{(2(K+1)-1)}^2}{3}=$$

$$\frac{(2K+1)[K(2K-1)+3(2K+1)]}{3}=$$

$$\frac{(2K+1)[2K^2-K+6K+3]}{3}=$$

$$\frac{(2K+1)(2K^2+5K+3)}{3} =$$

$$\frac{(2K+1)(2K+3)(K+1)}{3} =$$

WHICH IS WHAT WE NEEDED

BIG, PHAT, HENCE STMT...

Show, using PMI, that for all non-negative integers

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n}=2^n$$

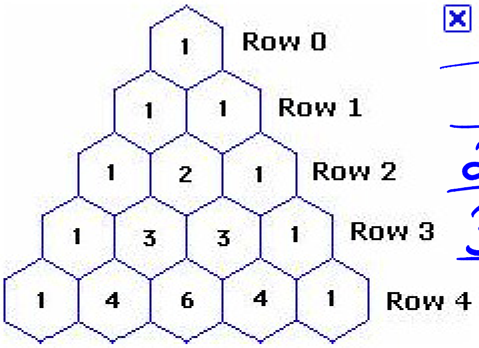
$n=0,1,2,3\ldots$

[Hey, it's Pascal's Triangle!]

Notice that this one is slightly different because were are not just considering $n \in Z^+$

Instead of $S(1)$ as our first step, we need to show that $S(0)$ is true as our first step.

Keep thinking about Pascal's Triangle.



Row	Sums
0	2^0
1	2^1
2	2^2
3	2^3
4	2^4

Notice that $\binom{3}{1}+\binom{3}{2}=\binom{4}{2}$

In general, $\binom{n}{r-1}+\binom{n}{r}=\binom{n+1}{r}$

We are going to need this!

Let S(n) be the statement that for n = 0, 1, 2, 3, …

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n}=2^n$$

$S(0) = \binom{0}{0} = 1 = 2^0$

so, S(0) is TRUE

Assume that S(k) is true for k = 1, 2, 3,…
Namely, that

$$\binom{K}{0}+\binom{K}{1}+\binom{K}{2}+\ldots+\binom{K}{K}=2^K$$

need To show That $S(K+1) = 2^{K+1}$

What do you get when you add:

$$\binom{K}{0}+\binom{K}{1}+\binom{K}{2}+\dots+\binom{K}{K-1}+\binom{K}{K}=2^K$$

☆ $$\binom{K}{0}+\binom{K}{1}+\binom{K}{2}+\dots+\binom{K}{K-1}+\binom{K}{K}=2^K$$

$$\binom{K}{0}+\binom{K+1}{1}+\binom{K+1}{2}+\dots+\binom{K+1}{K}+\binom{K}{K}=2\cdot 2^K$$

But, since $\binom{k}{0}=\binom{k+1}{0}=\binom{k}{k}=\binom{k+1}{k+1}=1$, then

we can rewrite this as:

$$S(K+1)=\binom{K+1}{0}+\binom{K+1}{1}+\binom{K+1}{2}+\dots+\binom{K+1}{K+1}$$

$$=2^{K+1}$$

Hence, blah, blah, blah, ...

Another one to ponder:
Show that in an arithmetic sequence where $a_n = a_{n-1} + d$, the n^{th} term can be given by the formula $a_n = a_1 + (n-1)d$

Let S(n) be the statement that for $n \in Z^+$ $a_n = a_{n-1} + d$ the n^{th} term can be given by the formula $a_n = a_1 + (n-1)d$

S(1) = $a_1 + (1-1)d = a_1$

AWESOME, IT'S TRUE

ASSUME S(K) is TRUE for $k \in \mathbb{Z}^+$

$$\boxed{a_k = a_1 + (k-1)d}$$

We need To SHOW

$$a_{k+1} = a_1 + (k+1-1)d$$
$$= \underline{a_1 + kd}$$

$$a_{K+1} = \boxed{a_K} + d$$
$$= \boxed{a_1 + (k-1)d} + d$$

$= a_1 + kd - d + d$

$= a_1 + kd$ VOILA!

Hence, blah, blah, blah, ...

Now do #2 and 4 on page 228 on your own

#2

Using PMI show that for $n \in Z^+$, $3^{2n+2} - 8n - 9$ is divisible by 64.

#4

Show that for $n = 0, 1, \ 2, \ 3, \ \ldots \ 5^n + 3$ is divisible by 4