**My IBHL Maths Revision for Complex Numbers** *ibhlcomplexrev.doc*

**Paper 1-type** *Instructions to candidates:*

*You are not permitted access to any calculator for these problems. Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Show all working, etc. Please do all of your working on separate pieces of paper.*

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|---|---|---|
| **1.5** | Complex numbers | $z = a + \mathrm{i}b = r(\cos\theta + \mathrm{i}\sin\theta) \;= r\mathrm{e}^{\mathrm{i}\theta} = r\,\mathrm{cis}\,\theta$ |
| **1.7** | De Moivre's theorem | $[r(\cos\theta + \mathrm{i}\sin\theta)]^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta) = r^n\mathrm{e}^{\mathrm{i}n\theta} = r^n\,\mathrm{cis}\,n\theta$ |

(1) **Paper 1 – type** M08/5/MATHL/HP1/ENG/TZ1/XX

Express $\dfrac{1}{(1-\mathrm{i}\sqrt{3})^3}$ in the form $\dfrac{a}{b}$ where $a, b \in \mathbb{Z}$.

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(2) **Paper 1 – type** M08/5/MATHL/HP1/ENG/TZ2/XX

**14.** *[Maximum mark: 12]*

Let $w = \cos\dfrac{2\pi}{5} + \mathrm{i}\sin\dfrac{2\pi}{5}$.

(a) Show that $w$ is a root of the equation $z^5 - 1 = 0$. *[3 marks]*

(b) Show that $(w-1)(w^4 + w^3 + w^2 + w + 1) = w^5 - 1$ and deduce that $w^4 + w^3 + w^2 + w + 1 = 0$. *[3 marks]*

(c) **Hence** show that $\cos\dfrac{2\pi}{5} + \cos\dfrac{4\pi}{5} = -\dfrac{1}{2}$. *[6 marks]*

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(3) **Paper 2 –type** M08/5/MATHL/HP2/ENG/TZ1/XX

[Go ahead and use that TI!]

Find, in its simplest form, the argument of $\left(\sin\theta + \mathrm{i}(1-\cos\theta)\right)^2$ where $\theta$ is an acute angle.

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(4) **Paper 2 – type** M08/5/MATHL/HP2/ENG/TZ2/XX

Consider $w = \frac{z}{z^2+1}$ where $z = x + \mathrm{i}y$, $y \neq 0$ and $z^2 + 1 \neq 0$.

Given that $\operatorname{Im} w = 0$, show that $|z| = 1$.

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(5) **Paper 1 – type** N07/5/MATHL/HP1/ENG/TZ0/XX+

Prove that $\left(\sqrt{3}+\mathrm{i}\right)^n + \left(\sqrt{3}-\mathrm{i}\right)^n$ is real, where $n \in \mathbb{Z}^+$.

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(6) **Paper 1 – type** N08/5/MATHL/HP1/ENG/TZ0/XX

**Part A** *[Maximum mark: 12]*

(a) Use de Moivre's theorem to find the roots of the equation $z^4 = 1 - \mathrm{i}$. *[6 marks]*

(b) Draw these roots on an Argand diagram. *[2 marks]*

(c) If $z_1$ is the root in the first quadrant and $z_2$ is the root in the second quadrant, find $\frac{z_2}{z_1}$ in the form $a + \mathrm{i}b$. *[4 marks]*

**Part B** *[Maximum mark: 13]*

(a) Expand and simplify $(x-1)(x^4 + x^3 + x^2 + x + 1)$. *[2 marks]*

(b) Given that $b$ is a root of the equation $z^5 - 1 = 0$ which does not lie on the real axis in the Argand diagram, show that $1 + b + b^2 + b^3 + b^4 = 0$. *[3 marks]*

(c) If $u = b + b^4$ and $v = b^2 + b^3$ show that

(i) $u + v = uv = -1$;

(ii) $u - v = \sqrt{5}$, given that $u - v > 0$. *[8 marks]*

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