CALCULUS REVISION #4

From our quiz:

$\int \left(e^{2x}\cos x\right) dx$

$u = e^{2x}$ $\quad$ $V = \sin x$

$du = 2e^{2x}\,dx$ $\quad$ $dv = \cos x\,dx$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\int e^{2x}\sin x\,dx$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\left[-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx\right]$$

$$5\int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x + C_1$$

1. Consider the function $y = \dfrac{3x-2}{x}$. The graph of this function has a vertical and a horizontal asymptote. [non-calculator]

a) Write down the equation of

i) the vertical asymptote $\quad x = 0$

ii) the horizontal asymptote $\quad y = 3$

b) Find $\frac{dy}{dx}$

$y = 3 - \dfrac{2}{x}$

$y' = \dfrac{2}{x^2}$

c) Indicate the intervals for which the curve is increasing or decreasing

$y' = \frac{2}{x^2}$

$y' > 0$ for all $x \neq 0$

y is INCREASING on the INTERVALS $]-\infty, 0[\cup]0, \infty[$

d) How many stationary points does the curve have? Explain fully.

NONE because $y' \neq 0$ FOR ANY VALUES OF $x \in \mathbb{R}$

2. The curve $y = ax^2 + bx + c$ has a maximum point at (2, 18) and passes through the point (0, 10). Find a, b, and c

$c = 10$

$y' = 2ax + b$ $\quad y'(2) = 0$

$0 = 4a + b$

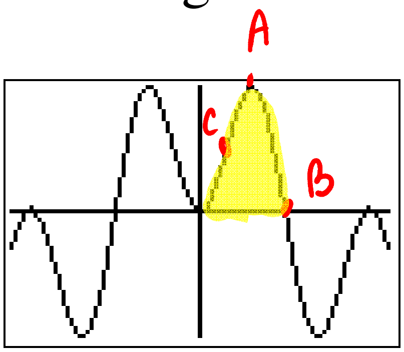
$18 = 4a + 2b + 10$

$a = -2, b = 8, c = 10$ ☺

3. [Calculator but must show all working]

The function f is given by $f(x) = (\sin x)^2 \cos x$

The diagram shows part of the graph of $y = f(x)$



$f(x) = \underline{(\sin x)^2}\ \underline{\cos x}$

$f'(x) = 2\sin x \cos^2 x + -\sin^3 x$

The point A is a maximum point, the point B lies on the x-axis, and the point C is a point of inflexion

a) Find $f'(x)$

$f'(x) = \sin x\,(2\cos^2 x - \sin^2 x)$

b) Hence, show that at the point A, $\cos x = \sqrt{\frac{1}{3}}$

OR $\cos x = \frac{1}{\sqrt{3}}$

$x \approx .95531662$ RADIANS

$f'(.95531662) = 0$

Hence at point A $\cos x = \frac{1}{\sqrt{3}}$

c) Find the exact maximum value

$f(.95531662) \approx .38490018$

OR $f(\arccos \frac{1}{\sqrt{3}}) = \frac{2\sqrt{3}}{9}$

d) Find the exact value of the x-coordinate at the point B

$f(x) = 0$

$x = \frac{\pi}{2}$

e) Find $\int f(x)\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int u^2\,du$

$$\int (\sin x)^2 \cos x \, dx$$

$$= \frac{\sin^3 x}{3} + C$$

f) Find the area of the shaded region [in yellow]

$$\int_0^{\frac{\pi}{2}} f(x)\,dx = \frac{1}{3}$$

g) Given that $f''(x) = 9(\cos x)^3 - 7\cos x$, find the x-coordinate at the point C

$f''(C) = 0$

$0 = 9(\cos^3 x) - 7\cos x$

$x = \arccos\left(\frac{\sqrt{7}}{3}\right)$

4. Solve the differential equation $x\frac{dy}{dx} - y^2 = 1$ given that $y = 0$ when $x = 2$. Give your answer in the form $y = f(x)$

5. Use the substitution $u = x + 2$ to find $\int \frac{x^3}{(x+2)^2}\, dx$