IBHL Maths Calculus Revision 3

Please note that these represent "Core" questions – not Paper 3-style questions. We will work on those later.

1.

Non-calculator

A function is define as $f(x) = 2x^3 - 3x^2 - 7x + 9$.

Find the equations of the two tangents to $f(x)$ that have a gradient of 5.

$f'(x) = 6x^2 - 6x - 7$

let $f'(x) = 5$

$5 = 6x^2 - 6x - 7$

$0 = 6x^2 - 6x - 12$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$f(-1) = 11$

$f(2) = -1$

EQUATIONS OF TAN lines

$y - 11 = 5(x+1)$

$y + 1 = 5(x-2)$

2.

Find the points at which the tangent is parallel to the y-axis of the curve $3x^2+2xy - y^2 - 4 = 0$.

$\frac{d}{dx}3x^2 + \frac{d}{dx}\underline{2xy} - \frac{d}{dx}y^2 - \frac{d}{dx}4 = \frac{d}{dx}0$

$6x + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x-2y) = -6x - 2y$

$\frac{dy}{dx} = \frac{-3x-y}{x-y}$

TO BE PARALLEL TO y-AXIS

$\frac{dy}{dx}$ must be undefined or

$x-y=0$

$(1,1)$ $3x^2+2xy-y^2-4=0$

$3+2-1-4=0$

let's let $x=y$ in the curve

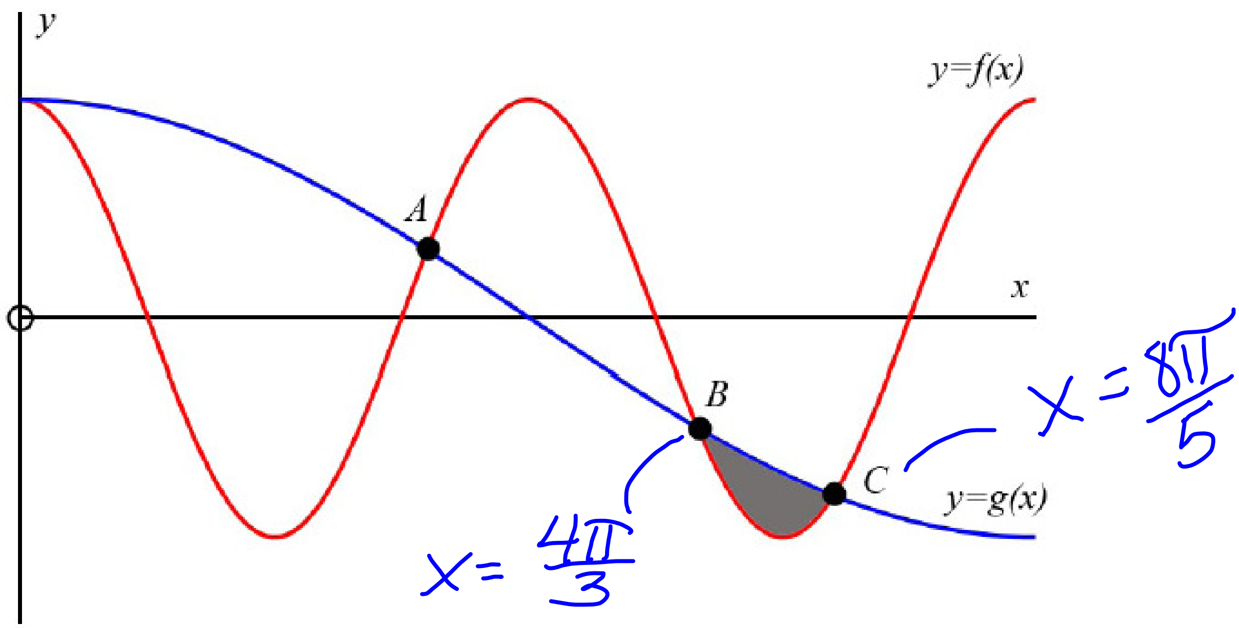
$$\left.\begin{aligned} 3y^2+2y^2-y^2-4&=0\\ 4y^2&=4\\ y&=\pm 1 \end{aligned}\right\}$$ we need TO SHOW this

$(-1,-1)$ $3+2-1-4=0$

3.

Calculator-friendly

The graph below shows two functions $f(x)$ and $g(x)$ where, $f(x) = \cos(2x)$ and $g(x) = \cos\left(\frac{1}{2}x\right)$ over the interval $0 \le x \le 2\pi$.



a) Show that the exact values of A, B and C are $\frac{4\pi}{5}$, $\frac{4\pi}{3}$ and $\frac{8\pi}{5}$ respectively.

[6 marks]

b) Write an expression for the shaded region. *[2 marks]*

c) Calculate the area of the shaded region, correct to 2 decimal places. *[4 marks]*

$$\text{Area}_{\text{shaded}} = \int_{\frac{4\pi}{3}}^{\frac{8\pi}{5}} \left[\cos\left(\tfrac{1}{2}x\right) - \cos(2x)\right] dx$$

$$\approx .17$$

4. Another Calculator question from a released exam:

Particle A moves in a straight line, starting from O_A, such that its velocity in metres per second for $0 \le t \le 9$ is given by $v_A = -\frac{1}{2}t^2 + 3t + \frac{3}{2}$

Particle B moves in a straight line, starting from O_B, such that its velocity in metres per second for $0 \le t \le 9$ by $v_B = e^{0.2t}$

(a) Find the maximum value of v_A, justifying that is a maximum. [A graph is not justification]

$\frac{d}{dt} V_A = -t + 3$

$0 = -t + 3$

$t = 3$

$\frac{d}{dt}(-t+3) = -1$

at $t = 3$, the second derivative of $V_A = -1$

Hence at $t = 3$, V_A has a max. value

$V_A(3) =$

(b) Find the acceleration of B when t = 4

$V'(t) = a(t)$

$= \frac{1}{5} e^{.2t}$

$a(4) \approx .445 \ \frac{m}{sec^2}$

The displacements of A and B from O_A, and O_B respectively, at time t are S_A metres and S_B metres. When $t = 0$, $S_A = 0$, and $S_B = 5$.

(c) Find an expression for S_A and S_B, giving your answer in terms of t.

$S_A = \int \left(-\frac{1}{2}t^2 + 3t + \frac{3}{2}\right) dt$

$= -\frac{1}{6}t^3 + \frac{3}{2}t^2 + \frac{3}{2}t + C$

$S_A(0) = 0$ Hence $C = 0$

$S_A = -\frac{1}{6}t^3 + \frac{3}{2}t^2 + \frac{3}{2}t$

(d) (i) Sketch the curves of S_A and S_B on the same diagram.

(ii) Find the values of t at which $S_A = S_B$

From our quiz:

$$\int \left(e^{2x} \cos x\right) dx$$