Calculus Friday

use PARTS

1. Find $\int e^x \cos x \, dx$

$u = e^x \quad v = \sin x$

$du = e^x dx \quad dv = \cos x \, dx$

$\int e^x \cos x \, dx$

$= e^x \sin x - \int \sin x \, e^x \, dx \qquad uv - \int v \, du$

$= e^x \sin x - \int \sin x \, e^x dx$

$u = e^x \quad v = -\cos x$

$du = e^x dx \quad dv = \sin x \, dx$

$\int e^x \cos x \, dx$

$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

$+ \int e^x \cos x \, dx$

$$\frac{2\int e^x \cos dx}{2} = \frac{e^x \sin x}{2} + \frac{e^x \cos x}{2} + C$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x \sin x + \frac{1}{2} e^x \cos x + C_1$$

2. Consider the function $f(x) = \ln\sqrt{x^2+4}$

(a) Find $f'(x)$ $f(x) = \ln[(x^2+4)^{1/2}]$

$f'(x) = \frac{1}{\sqrt{x^2+4}} \cdot \frac{1}{\cancel{2}}(x^2+4)^{-1/2} \cdot \cancel{2}x$

$= \frac{x}{x^2+4}$

(b) Find $f''(x)$

$f'(x) = \frac{x}{x^2+4} = x\,(x^2+4)^{-1} = \frac{1}{x^2+4} - \frac{2x^2}{(x^2+4)^2}$

3.

(a) Given that $\dfrac{x^2}{(1+x)(1+x^2)} = \dfrac{a}{(1+x)} + \dfrac{bx+c}{(1+x^2)}$

calculate the value of a, b, **and** c

START BY MULTIPLYING by the L.C.D. ☺

$x^2 = a(1+x^2) + (bx+c)(1+x)$

$x^2 : a + ax^2 + bx + bx^2 + c + cx$

$x^2 + 0x + 0 = x^2(A+B) + x(B+C) + A + C$

$A+B=1 \quad B=1-A$

$B+C=0 \quad 1-A+C=0,\ 1-2A=0$

$A+C=0 \quad C=-A$

$\boxed{A=\frac{1}{2}}$

$A+B=1,\ \boxed{B=\frac{1}{2}}$

$A+C=0,\ \boxed{C=-\frac{1}{2}}$

(b) Hence, find $I = \displaystyle\int \frac{x^2}{(1+x)(1+x^2)}\,dx$

(c) If $I = \frac{\pi}{4}$ **when** $x = 1$**, calculate the value of the constant of integration giving your answer in the form** $p + q\ln r$ **where** $p,\ q,\ r \in R$

4. Show, using first principles, that if $f(x) = \sin x$**, then** $f'(x) = \cos x$

Recall that $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

[This is "IB-speak" for first principles]