

1.2 Exponents and logarithms

$$a^x = b \Leftrightarrow x = \log_a b$$

$$a^x = e^{x \ln a}$$

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

1.3 Combinations

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Binomial theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

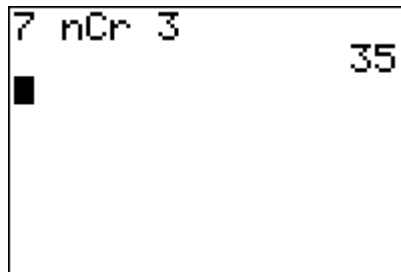
“First Principles”

Derivative of $f(x)$

$$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

TI Tip

How to find $\binom{n}{r}$ for example $\binom{7}{3}$



November 2005 TZ0 Paper 2 #4

- (a) Write down the term in x^r in the expansion of $(x+h)^n$, where $0 \leq r \leq n$, $n \in \mathbb{Z}^+$
- (b) Hence, differentiate x^n , $n \in \mathbb{Z}^+$, from first principles

May 2005 TZ1 Paper 1 #2 [You can use your TI but you must show your working]

Find the coefficient of x in the expansion of $\left(3x - \frac{2}{x}\right)^5$



May 2005 TZ1 Paper 1 #15 [non-calculator]

Solve the equation $2\log_3(x-3) + \log_{\frac{1}{3}}(x+1) = 2$



May 2007 TZ1 Paper 1 #19 [non-calculator]

The solution of $2^{2x+3} = 2^{x+1} + 3$ can be expressed in the form $a + \log_2 b$, where $a, b \in \mathbb{Z}$

Find the value of a and b



May 2008 TZ1 Paper 1 #1 [You can use your TI but you must show your working]

Determine the first three terms in the expansion of $(1-2x)^5(1+x)^7$ in ascending powers of x



Non-calculator

Solve for x in each of the following equations

(a) $\log_2 x^4 = \log_2 16.$

(b) $\log_x 27 = 3.$

(c) $\log_4 32 = x.$

(d) $\log_3(1-x) - \log_3 x = 2.$



Solve the following equations for x .

- (a) $x - 4 = 4$.
- (b) $|x - 4| = 4$.
- (c) $\log_2(x - 4) = 4$.
- (d) $x^2 - 4 = 4$.
- (e) $\log_2(x - 2) + \log_2(x + 2) = 2$.



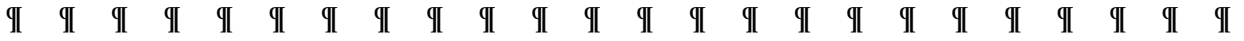
- (a) (i) Factorise the quadratic expression $x^2 - 6x + 5$.
- (ii) Solve the inequality for x , where $x^2 - 6x + 5 > 0$.
- (b) For what values of x is
 - (i) $\log_{10}(6 - x)$ defined?
 - (ii) $\log_{10}(x - 4)$ defined?
- (c) Solve for x , the equation $\log_{10}(x^2 - 6x + 5) = \log_{10}(6 - x) + \log_{10}(x - 4)$.



- (a) (i) Expand $(x + a)(x + 1)$.
- (ii) Factorise $x^2 + 3x + 2$.
- (b) Solve each of the following equations for x ,
 - (i) $x^2 - (e + 1)x + e = 0$.
 - (ii) $e^{2x} - (e + 1)e^x + e = 0$.



4. (5 points) Determine the term independent of x in the expansion of $\left(2x^3 - \frac{1}{x}\right)^8$.



Find the constant term in the expansion $\left(2x + \frac{1}{x}\right)^{2n}$.



- (a) Determine the maximal domain of $f(x) = \ln(x + 1)$.
- (b) If $g(x) = e^x$,
 - (i) justify the existence of $(f \circ g)(x)$.
 - (ii) find $(f \circ g)(x)$.
 - (iii) state the maximal domain of $(f \circ g)(x)$.



The graph of $f(x) = -1 + \ln(x - 1)$, $x > a$ is shown.

- (a) Find the values of a and b .
- (b) Find $f^{-1}(x)$.
- (c) Sketch the graph of $f^{-1}(x)$.

