My IBHL [Brief] Algebra Revision

Core questions tend to focus on quadratics [axis of symmetry, vertex form, discriminant, transformation of graphs, domain, range, and for some curious reason – inverses]

If $f(x) = ax^2 + bx + c$, then the axis of symmetry is $x = \frac{-b}{2a}$

The discriminant of a quadratic equation is $b^2 - 4ac$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{for } a \neq 0$$

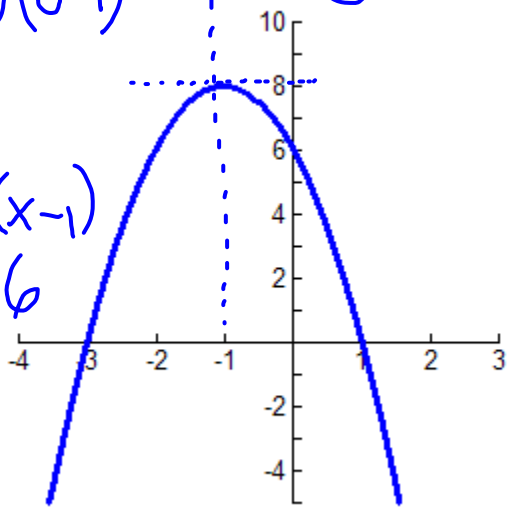
Let's look at some Power Points that I purchased

Ibhlquadratics
Ibhldiscriminant
Ibhlcompletesquare
Ibhlcompletesquare2

Let's try just a few questions:
Find the equation of each of the following quadratic functions in the form
$f(x) = a(x-p)(x-q)$ and also
$f(x) = ax^2 + bx + c$

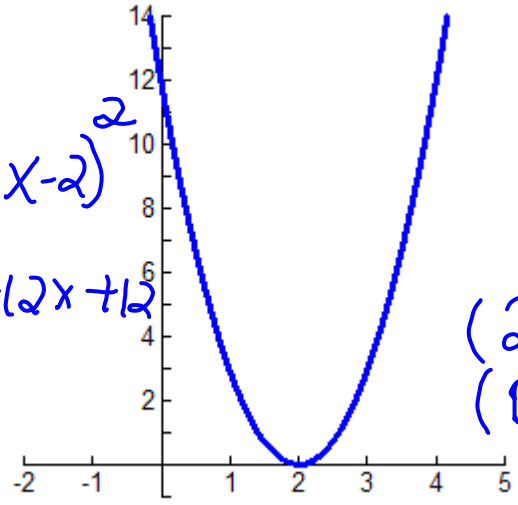
$6 = a(0-(-3))(0-1)$
$6 = -3a$
$-2 = a$
$f(x) = -2(x+3)(x-1)$
$f(x) = -2x^2 - 4x + 6$

$x = -1$ axis of symmetry



$(-3,0)$
$(1,0)$
$(0,6)$

$f(x) = 3(x-2)^2$
$f(x) = 3x^2 - 12x + 12$



$(2,0)$
$(0,12)$

Here is a table that shows the properties of quadratics

Quadratic function, $a \neq 0$	**About the graph of the function**	**Results**
General form $f(x) = ax^2 + bx + c$ $\Delta = b^2 - 4ac$ [discriminant]	Parabola opens up if $a > 0$ Parabola opens down if $a < 0$ If $\Delta \geq 0$, then f has x-intercepts If $\Delta < 0$, then f has no intercepts	Axis of symmetry is $x = \frac{-b}{2a}$ If $\Delta \geq 0$, f has intercept(s) $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, 0 Vertex is: $\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$
Vertex Form $f(x) = a(x-h)^2 + k$	Vertex is at the point (h, k)	Axis of symmetry is $x = h$
Factorized form for two distinct rational zeros $f(x) = a(x-p)(x-q)$	Vertex is at $\left(\frac{p+q}{2}, f\left(\frac{p+q}{2}\right)\right)$	Axis of symmetry is $x = \frac{p+q}{2}$ ***X***-intercepts are $(p,0), (q,0)$
Factorized form for one rational root $f(x) = a(x-p)^2$	Vertex is at $(p, 0)$	Axis of symmetry is $x = p$ Vertex = x-intercept = $(p,0)$

Problems to ponder:

Find all possible values of m so that the graph of the function $g : x \mapsto mx^2 + 6x + m$ does not touch the x-axis

$\Delta = 36 - 4m^2$

$0 = 36 - 4m^2$

$m^2 = 9$

$m = \pm 3$

$m < -3$ or $m > 3$

Find the value(s) of p for which the equation $2x^2 + px + 1 = 0$ has one real solution.

let $\Delta = 0$

$0 = p^2 - 8$

$8 = p^2$

$\pm 2\sqrt{2} = p$

The equation $x^2 - 4kx + 4 = 0$ has two distinct real solutions. Find the set of all possible values of k.

WILL FINISH THURSDAY

The maximum value of the function $ax^2 + bx + c$ is 10. Given that $f(3) = f(-1) = 2$, find $f(2)$