Let's finish our vector problem:

20 October 2010

Note: Next week's quiz will cover complex numbers and vectors.

(c) The point $Q(3, 4, 3)$ lies on Π. The line L passes through the midpoint of $[PQ]$. Point S is on L such that $\left|\vec{PS}\right| = \left|\vec{QS}\right| = 3$, and the triangle PQS is normal to the plane Π. Given that there are two possible positions for S, find their coordinates.

at S $|PR| = 3$

$$(1+2\lambda)^2 + \left(\frac{5}{2} - \lambda\right)^2 + \left(\frac{1}{2}+\lambda\right)^2 = 9$$

because $PR = PO + OR$

$$= \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 + 2\lambda \\ \frac{3}{2} - \lambda \\ \frac{5}{2} + \lambda \end{pmatrix}$$

$$6\lambda^2 = \frac{6}{4}$$

$$\lambda = \pm\frac{1}{2}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{3}{2} \\ \frac{5}{2} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$S_1 = (3, 1, 3)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{3}{2} \\ \frac{5}{2} \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$S_2 = (1, 2, 2)$$

And now let's finish our Calculus problems from two weeks ago. [I can't sleep until we finish them]

5. More trig stuff [u-sub]

Find $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\sqrt{\cos x}}\, dx$.

$$u = \cos x$$
$$du = -\sin x\, dx$$
$$-du = \sin x\, dx$$
$$u(0) = 1$$
$$u\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$= -\int_1^{\frac{\sqrt{2}}{2}} u^{-\frac{1}{2}}\, du$$

$$= \int_{\frac{\sqrt{2}}{2}}^{1} u^{-\frac{1}{2}}\, du$$

$$= 2\sqrt{u}\,\Big|_{\frac{\sqrt{2}}{2}}^{1}$$

$$= 2\left(1 - 2^{-\frac{1}{4}}\right)$$
$$= 2 - 2^{\frac{3}{4}}$$

6. If time [this is a non-calculator problem]

It is given that

$$f(x) = \frac{18(x-1)}{x^2},\ \ f'(x) = \frac{18(2-x)}{x^3},\ \text{and}\ f''(x) = \frac{36(x-3)}{x^4},\ \ x \in \mathbb{R},\ x \neq 0.$$

(a) Find

(i) the zero(s) of $f(x)$; $f(1) = 0$ at $x = 1$

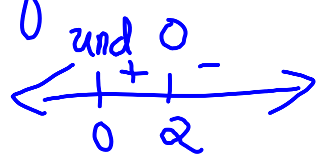
(ii) the equations of the asymptotes; $\lim\limits_{x\to 0} f(x) = -\infty$, $x = 0$ VERTICAL Asymptote; $\lim\limits_{x\to\infty} \frac{18x-18}{x^2} = 0$, $y = 0$ hor. Asymp.

(iii) the coordinates of the local maximum and justify it is a maximum;

(iv) the interval(s) where $f(x)$ is concave up.

(b) Hence sketch the graph of $y = f(x)$.

$$f'(x) = \frac{18(2-x)}{x^3} \quad \text{for } x \neq 0$$



$f'(2) = 0$

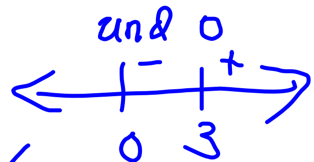
at $x = 2$ $f'(x)$ changes from POSITIVE VALUES

Hence f has a max at $x = 2$

or $f''(2) < 0$ Hence f has a max at $x = 2$

By SECOND DERIVATIVE TEST

$f'''(3) = 0$



Since $f''(x) > 0$ on $(3, \infty)$
then f is c. up on this interval