FTC in action!
Notes from 1/9/9

The infamous Sandy Point Beach Problem:
The following problem is the property of ETS College Board: [calculator-friendly]

2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

2. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

$$\int_0^6 R(t)\,dt \approx 31.816 \text{ yd}^3$$
OR 31.815 yd^3

$y(t)$ = AMOUNT OF SAND ON BEACH at time t

$$y(t) = 2500 - \int_0^t R(x)\,dx + \int_0^t S(x)\,dx$$

2500: INITIAL AMOUNT
$\int_0^t R(x)\,dx$: AMOUNT REMOVED
$\int_0^t S(x)\,dx$: AMOUNT PUMPED IN

we need $y'(4)$

$$y'(t) = \frac{d}{dt}\left[2500 - \int_0^t R(x)\,dx + \int_0^t S(x)\,dx\right]$$

$$y'(t) = -R(t) + S(t)$$

or $S(t) - R(t)$

$$y'(4) = S(4) - R(4)$$

$$y'(4) \approx -1.909 \frac{yd^3}{hr} \text{ or } -1.908 \frac{yd^3}{hr}$$

At $t \approx 5.117865$ $y'(t)$ changes from negative to positive values,

we will check $y(0)$, $y(5.117865)$, $y(6)$

$$y(0) = 2500 - \int_0^0 R(x)\,dx + \int_0^0 S(x)\,dx$$

$$y(0) = 2500\ yd^3$$

$$y(5.117865) = 2500 - \int_0^{5.117865} R(x)\,dx + \int_0^{5.117865} S(x)\,dx$$

$$y(5.117865) = 2492.3694$$

$$y(6) = 2500 - \int_0^6 R(x)\,dx + \int_0^6 S(x)\,dx$$

$$y(6) \approx 2493.2766$$

The AMT of SAND is a minimum when $t \approx 5.117865$ and the min. amt is ≈ 2493.2766 yd^3

Tonight's [01/09/09] homework problem:
Consider the ***Camp Newton Problem*** 2005AB2B
The following problem is the property of ETS College Board

Sir Isaac [not Fig] Newton

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

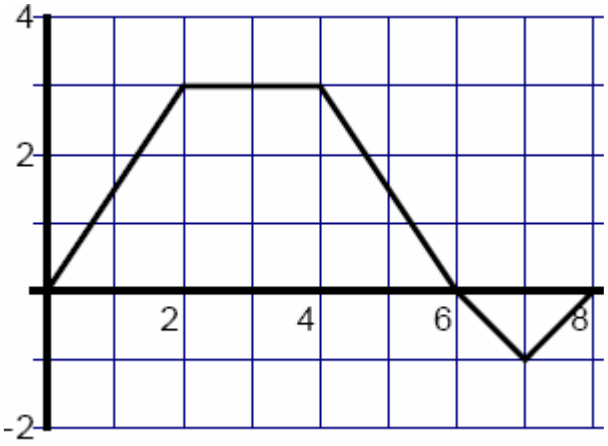
(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

Notes from 1/12/09
Graph from Mr. Zab's webpage:



The graph of the function f shown above consists of five line segments. Let g be the function given by

$$g(x) = \int_0^x f(t)\, dt$$

Find $g(0)$, $g(6)$, $g(8)$

Find $g'(0)$, $g'(3)$, $g'(7)$, $g'(8)$

Find all values of x on the open interval at which g attains a relative maximum.

Find the absolute minimum value of g.

Find all values of x in the open interval at which the graph of g has a point of inflection.