

Definition of the Derivative [the Limit Definition]

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$

Examples:

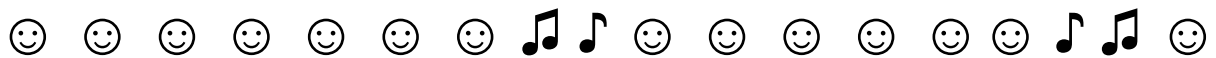
$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \frac{d}{dx}(x^2) = 2x$$

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \frac{d}{dx}(x^2) \Big|_{x=5} = 2x \Big|_{x=5} = 10$$

$$\lim_{h \rightarrow 0} \frac{(5+h)^2 - 25}{h} = \frac{d}{dx}(x^2) \Big|_{x=5} = 2x \Big|_{x=5} = 10$$

The $\lim_{h \rightarrow 0} \frac{|x+h| - |x|}{h}$ at $x=3$ is

- (A) -1
- (B)
- (C) 1
- (D) 3
- (E) nonexistent



If f is a function such that $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = 0$, then which of the following must be true?

- (A) $\lim_{x \rightarrow a} f(x)$ does not exist
- (B) $f(a)$ does not exist
- (C) $f(a) = 0$
- (D) $f'(a) = 0$
- (E) $f(x)$ is continuous at $x = 0$

$$\lim_{h \rightarrow 0} \frac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h} \text{ is}$$

- (A) 0
- (B) $10x^3 - 15x$
- (C) $10x^4 + 15x^2$
- (D) $-10x^4 + 15x^2$
- (E) $10x^4 - 15x^2$



What is $\lim_{h \rightarrow 0} \frac{\cos(\pi + h) - \cos(\pi)}{h}$?

- (A) 0
- (B) -1
- (C) 1
- (D) $\frac{1}{\sqrt{2}}$
- (E) The limit does not exist