Definition of the Derivative [the Limit Definition]

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)$$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=f'(a)$$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=f'(a)$$

Examples:

$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=\frac{d}{dx}\left(x^2\right)=2x$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=\frac{d}{dx}\left(x^2\right)\Big|_{x=5}=2x\Big|_{x=5}=10$$

$$\lim_{h\to 0}\frac{(5+h)^2-25}{h}=\frac{d}{dx}\left(x^2\right)\Big|_{x=5}=2x\Big|_{x=5}=10$$

The $\lim_{h\to 0} \frac{|x+h|-|x|}{h}$ at x=3 is

(A) -1

(B)

(C) 1

(D) 3

(E) nonexistent

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If f is a function such that $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = 0$, then which of the following must be true?

(A) $\lim_{x\to a} f(x)$ does not exist

(B) $f(a)$ does not exist

(C) $f(a) = 0$

(D) $f'(a) = 0$

(E) $f(x)$ is continuous at x = 0

$$\lim_{h \to 0} \frac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h} \text{ is}$$

(A) 0

(B) $10x^3 - 15x$

(C) $10x^4 + 15x^2$

(D) $-10x^4 + 15x^2$

(E) $10x^4 - 15x^2$

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What is $\lim_{h \to 0} \frac{\cos(\pi + h) - \cos(\pi)}{h}$?

(A) 0

(B) -1

(C) 1

(D) $\frac{1}{\sqrt{2}}$

(E) The limit does not exist