EVERYTHING I AM SUPPOSED TO REMEMBER ABOUT DERIVATIVES

The basic concept: a derivative is a rate of change!!! Finding the derivative function will give us the slopes of the tangents to the curve at any point.

The average rate of change is the slope of the secant between the endpoints of the curve.

Average rate of change $= \dfrac{f(b)-f(a)}{b-a} = m_{\text{sec}ant}$

Instantaneous rate of change =

$\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = f'(x)$ **=** $m_{\tan gent}$

Let f be differentiable for $a < x < b$ and continuous for $a \le x \le b$

If $f'(x) > 0$ for every x in (a, b), then f is increasing on [a, b].

If $f'(x) < 0$ for every x in (a, b), then f is decreasing on [a, b].

If $f'(c)=0$, then $f(x)$ has a horizontal tangent at x = c.

Suppose that $f''(x)$ exists on the interval (a, b).
If $f''(x)>0$ for every x in (a, b), then f is concave up on (a, b)
OR
If $f'(x)$ is increasing for every x in (a, b), then f is concave up on (a, b)

If $f''(x)<0$ for every x in (a, b), then f is concave up on (a, b)
OR
If $f'(x)$ is decreasing for every x in (a, b), then f is concave down on (a, b)

Extreme Value Theorem
If f is continuous on a closed interval [a, b], then $f(x)$ has both a maximum and a minimum value on [a, b]

To find the maximum and minimum values of a function, $y = f(x)$, locate
1. the point(s) where $f'(x)$ changes sign [commonly known as the critical values or critical numbers]. These are the values where either $f'(x)=0$ or is undefined.

2. the endpoints, if any, on the domain of $f(x)$

Compare the function values at all of these points to find the maximums and minimums.

♪ Make sure that you are answering the question. Do you want where the extrema occur [i.e. the x-value] or what is the value of the extrema [i.e. the y-value]

Rolle's Theorem

If f is continuous on [a, b] and differentiable on (a, b) such that $f(a) = f(b)$, then there is at least one number c in the open interval (a,b) where $a < c < b$ such that $f'(c) = 0$

Mean Value Theorem

If f is continuous on [a, b] and differentiable on (a, b), then there is at least one number c, where a < c < b, such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

In other words, there is some place where the instantaneous rate of change must equal the average rate of change.

Inverse functions

If f is differentiable at every point on an interval I and $f'(x) \neq 0$ on I, then $g = f^{-1}(x)$ is differentiable at every point of the interior of the interval $f(I)$ AND

$$g'(f(x)) = \frac{1}{f'(x)}$$

If (4, 5) is a point on the graph of $y = f(x)$ which has an inverse function and $f'(4) = 2$, then what must be true?

IF (4,5) is A POINT ON $y = f(x)$, then WHAT POINT MUST BE ON the INVERSE GRAPH? (5,4)

then ACCORDING TO OUR RULE

$$g'(5) = \frac{1}{f'(4)}$$

Hence, $g'(5) = \frac{1}{2}$

If $f(x) = 2 + \frac{4}{x}$ and g is the inverse of f, then $g'(10) =$

FIRST LET'S FIND THE VALUE OF X WHICH MAKES $f(x) = 10$ [we need the point]

$10 = 2 + \frac{4}{x}$ $\quad x = \frac{1}{2}$

so on $f(x)$ WE HAVE THE POINT $(\frac{1}{2}, 10)$ WHICH MEANS ON $g(x) = f^{-1}(x)$ we HAVE the POINT $(10, \frac{1}{2})$

NOW LET'S FIND $f'(x)$

$f'(x) = -\frac{4}{x^2}$ or $-4x^{-2}$

AND FIND $f'(\frac{1}{2}) = -16$

NOW USE OUR HANDY-DANDY RULE

$$g'(10) = \frac{1}{f'(\frac{1}{2})} = -\frac{1}{16}$$