

AP Worksheet #7
End of Chapter 5

Due Date: _____

Score: _____

Name: _____

Grading Scale
(61 Points Possible)

Section I – No calculators (Please show all work)	100%	= 56 correct	68%	= 38 correct
	95%	= 54 correct	65%	= 36 correct
	90%	= 52 correct	60%	= 34 correct
	88%	= 50 correct	58%	= 32 correct
	85%	= 48 correct	55%	= 30 correct
	80%	= 46 correct	50%	= 28 correct
	78%	= 44 correct	48%	= 26 correct
	75%	= 42 correct	45%	= 24 correct
	70%	= 40 correct	40%	= 22 correct

1. If $f(x) = 5x^{3/4}$, then $(f^{-1})'(5) =$ _____

2. $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{4x^2 + 2x + 5} =$ _____

3. If $f(x) = \frac{3x^2 + x}{3x^2 - x}$ then $f'(x)$ is _____ (write as a single fraction)

4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$ _____

5. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$ _____ (write as a single fraction)

6. If $f(x) = \sec x + \csc x$, then $f'(x) =$ _____

7. An equation of a line normal to the graph of $y = \sqrt{3x^2 + 2x}$ at $(2, 4)$ is _____

8. $\int_{-1}^1 \frac{4}{1+x^2} dx =$ _____

9. If $f(x) = \cos^2 x$, then $f''(\pi) =$ _____

10. If $f(x) = \frac{5}{x^2 + 1}$ and $g(x) = 3x$, then $g(f(2)) =$ _____

11. $\int x\sqrt{5x^2 - 4} dx =$ _____ (write using rational exponents)

12. The slope of the line tangent to the graph $3x^2 + 5 \ln y = 12$ at $(2, 1)$ is _____

13. The equation $y = 2 - 3 \sin \frac{\pi}{4}(x-1)$ has a fundamental period of _____

14. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum? _____

15. If $f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 7x - 5 & \text{if } x \geq 2 \end{cases}$, for all real numbers x , which of the following must be true? Justify

- I. $f(x)$ is continuous everywhere.
- II. $f(x)$ is differentiable everywhere.
- III. $f(x)$ has a local minimum at $x = 2$.

- (A) I only (B) I and II only (C) II and III only
(D) I and III only (E) I, II, and III

16. The acceleration of a particle moving along the y -axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at $t =$ _____

17. The average value of a function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is _____

18. If $f(x) = \sqrt{(x^3 + 5x + 121)}(x^2 + x + 11)$ then $f'(0) =$ _____

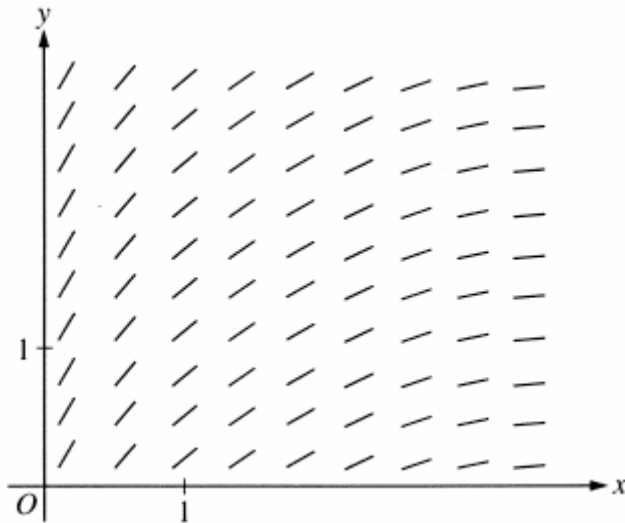
19. $\lim_{x \rightarrow 0} 4 \frac{\sin x \cos x - \sin x}{x^2} =$ _____

20. If $f(x) = \cos^3(x+1)$, then $f'(\pi) =$ _____

21. $\int x\sqrt{x+3} dx =$ _____

22. If $f(x) = \ln(\ln(1-x))$, then $f'(x) =$ _____

23.



The slope field for a certain differential equation is shown to the right.

Which of the following could be a specific solution to that differential equation? Explain your reasoning.

- A. $y = x^2$
- B. $y = e^x$
- C. $y = e^{-x}$
- D. $y = \ln x$

24. If F' is a continuous function for all real x , then $\lim_{h \rightarrow 0} \frac{1}{h} \int_a^{a+h} F'(x) dx =$ _____

Section II (calculators may be used)

25. $\int_0^{\pi/4} \sin x dx + \int_{-\pi/4}^0 \cos x dx =$ _____

26. Boats A and B leave the same place at the same time. Boat A heads due North at 12 km/hr. Boat B heads due East at 18 km/hr. After 2.5 hours, how fast is the distance between the boats increasing (in km/hr)? _____

27. $\lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{6} + h\right) - \tan\left(\frac{\pi}{6}\right)}{h} = \underline{\hspace{2cm}}$

28. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx = \underline{\hspace{2cm}}$

29. if $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be $\underline{\hspace{2cm}}$

30. The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum at $\underline{\hspace{2cm}}$

31. The average value of the function $f(x) = \ln^2 x$ on the interval $[2,4]$ is $\underline{\hspace{2cm}}$

32. $\frac{d}{dx} \int_0^{3x} \cos(t)dt = \underline{\hspace{2cm}}$

33. If the definite integral, $\int_1^3 (x^2 + 1)dx$, is approximated by using the Trapezoid Rule when $n = 4$, the error is $\underline{\hspace{2cm}}$.

34. $\frac{d}{dx} \left[\int_1^{x^2-3} \ln(2t) dt \right] = \underline{\hspace{2cm}}$

35. If the function $f(x)$ is continuous and differentiable $f(x) = \begin{cases} ax^3 - 6x; & \text{if } x \leq 1 \\ bx^2 + 4; & x > 1 \end{cases}$ then $a = \underline{\hspace{2cm}}$

36. Two particles leave the origin at the same time and move along the y-axis and their respective position determined by the functions $y_1 = \cos 2t$ and $y_2 = 4 \sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration? $\underline{\hspace{2cm}}$

37. Find the distance traveled (to thee decimal places) in the first 4 seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$; where t stands for time. $\underline{\hspace{2cm}}$

38. $\int \tan^6 x \sec^2 x dx = \underline{\hspace{2cm}}$

Section III Free Response Questions (No calculator)

1. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.

- (a) Find an equation of the line tangent to the graph at $(2,17)$
- (b) Find the x - and y -coordinates of the relative minima and relative maxima. Justify.
- (c) Find the x - and y -coordinates of the points of inflection. Justify.

2. A particle moves along the x -axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position $x(1) = 9$.
- (a) Write an expression for the velocity of the particle $v(t)$.
 - (b) At what values of t does the particle change direction? Justify.
 - (c) Write an expression for the $x(t)$ of the particle.
 - (d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

3. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Sketch a solution curve that passes through the point $(0, 1)$ on your slope field.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(d) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(e) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

Section IV Free Response (calculator may be used)

4. Sea grass grows (in tons) on a lake and the rate of growth of the sea grass is proportional to the sea grass.
 - (a.) Find an expression for G , the amount of sea grass in the lake (in tons), in terms of t , the number of years, if the amount of grass is 100 tons initially, and 120 tons after one year.
 - (b.) In how many years will the amount of sea grass available be 300 tons?
 - (c.) If fish are now introduced into the lake and consume a constant 80 tons/year of sea grass, how long will it take the lake to be completely free of sea grass?

5. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$

(a) Find $\frac{dy}{dx}$.

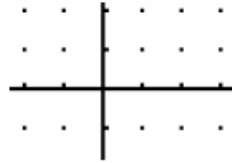
(b) Write an equation for the line tangent to the curve at the point $(4, -1)$

(c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k .

(d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve and solve for k .

6. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, 1)$. Then use your tangent line equation to estimate the value of $f(1.2)$

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(d) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part (c). Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.