Section 2.1 – The Derivative and the Tangent Line Problem

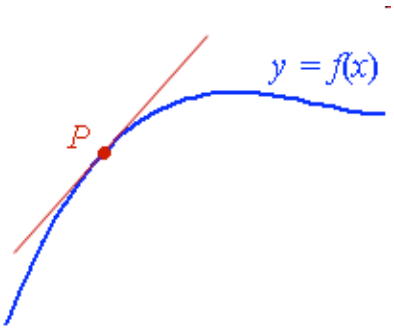
Objectives:

1. To estimate the slope of a curve at an indicated point.
2. To use the definition of the derivative to find the slope of the tangent line to a point on a curve and determine the equation of the tangent line.

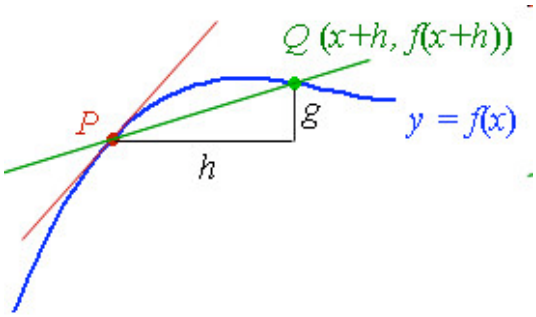
I. Tangent Line Problem

A. How do we find the equation of the tangent line?

B. Essentially, the problem of finding tangent line at a point *P* boils down to the problem of finding the slope of the tangent line at point *P*



C. Can we approximate the slope using a secant line which contains the points *P* and *Q*? How will this help?



D. Slope of Tangent Line

1. Gives an equation:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \text{\_\_\_\_\_}$$

2. Gives just a value:

$$\lim_{x \to c} \text{\_\_\_\_\_}$$

E. Examples:

1. Find an equation that will find all the tangent line slopes to the graph $f(x) = x^2 + 1$

2. Find the slope of the tangent line to the graph of $f(x) = x^2 - 2x + 1$ at the point (2,1). Use alternate method.

II. Derivative of a Function (Rate of Change at a given moment in time)

A. $f'(x) =$

The derivative is a function that gives the slope of the tangent line to the graph of f at the point $(x, f(x))$, provided that the graph has a tangent line at that point.

B. Symbols:

C. Examples

1. Find $f'(x)$ if $f(x)=\frac{1}{x}$

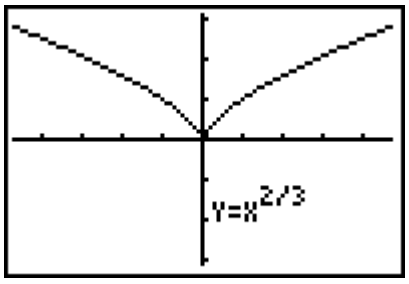
2. Find $\frac{dy}{dx}$ if $y=\cos x$

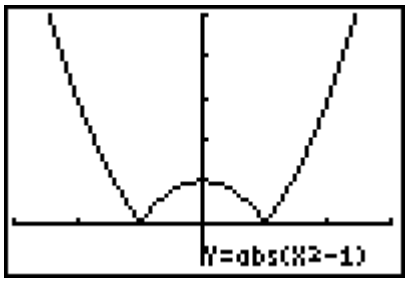
3. Find an equation of the tangent line to the function at the given point and verify on GC.

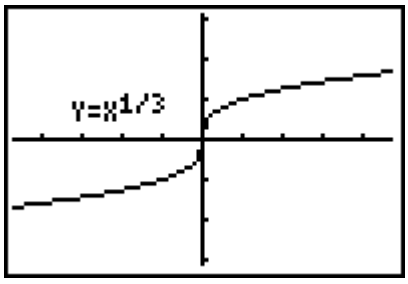
$f(t)=\sqrt{t};\ (4,2)$

D. _____ Implies _____

1. If *f* has a _____ at $x=c$, then *f* is _____ at *c*.
2. Notes:
   a) It is possible for a function to be _____ at $x=c$ and not be _____ at $x=c$.
   b) Continuity _____ _____ imply _____.
   c) Examples:







E. A function has a _____ at $x=c$, iff the function's _____-hand and _____-hand derivatives are _____ and _____.

Example

$$f(x)=\begin{cases}x^2, & x\le 0\\ 2x, & x>0\end{cases}$$

Does it have a derivative at $x=0$?

Homework: p.103 – 1, 4, 5, 9, 10, 11, 17, 21, 26, 27, 33, 39-48 all, 51, 61, 64, 74, 75, 83-88 all, 96, 97

Section 2.2 – Basic Differentiation Rules and Rates of Change

Objectives:

1. To use the rules of differentiation to calculate derivatives
2. To apply derivatives to rates of change

I. Basic Differential

A. Rules of Differentiation

1. The Constant Rule:
2. The Power Rule:
3. The Constant Multiple Rule:
4. Sum and Difference Rule:
5. Sine and Cosine Functions:
   a)
   b)

B. Examples

1. $y = 5$
2. $f(x) = 3x^5 - 2x^2 + 1$
3. $f(x) = (x-2)^2$
4. $y = \dfrac{5}{x^2}$
5. $f(x) = 3\sqrt[5]{x^2}$
6. $f(x) = 5\sin x$
7. Evaluate the derivative of the function at the given point.
   a) $f(x) = 2x^2 - x - 1; \quad (2,5)$
   b) $s(t) = -16t^2 + 16t + 32; \quad (2,0)$
8. Find the equation of the tangent line to the curve $y = 3x - x^3$ when $x = 2$.
9. Find the horizontal tangent lines for the function $f(x) = -x^2 + 6x + 5$

10. In the following functions graph and stated the intervals on which the function has positive slopes and negative slopes.

a) $y = x^3$

b) $f(x) = \frac{1}{x^2}$

c) $y = \sqrt[3]{x}$

II. Rates of Change

A. The derivative tells us the rate of change of one quantity compared to another at a particular instant or point (so we call it "instantaneous rate of change"). This concept has many applications in electricity, dynamics, economics, fluid flow, population modeling, and so on.

B. We learned before that velocity is distance divided by time. But this only works if the velocity is constant. We need a new method if the velocity is changing all the time. Rates of Change

C. Information

1. Average Velocity:
2. Instantaneous Velocity:
3. The derivative tells us:
    a) the rate of change of one quantity compared to another.
    b) the slope of a tangent to a curve at any point.
    c) the velocity if we know the expression s, for displacement:
    d) the acceleration if we know the expression v, for velocity:

D. Example

At time, $t = 0$, a diver jumps from a diving board that is 32-ft above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$ where *s* is measured in feet and *t* is measured in seconds.

a) Find the average velocity of the diver over the time interval of [1,1.2] seconds

b) When does the diver hit the water? What is the diver's velocity at impact?

c) During what interval is the diver moving upward?

d) When does the diver reach a maximum height? What is the maximum height?

HW: p.113 – 2, (3-51)/3, 56, 57-63 odds, 65, 72-76 all, 87-92 all, 97, 101-104 all, 107, 115, 117, 118

Section 2.3 – The Product and Quotient Rules and Higher-Order Derivatives

Objectives:

1. To use the rules of differentiation to calculate derivatives
2. To apply derivatives to higher –order derivatives

I. Basic Differential

A. More Rules of Differentiation

1. The Product Rule: $\frac{d}{dx}(u \cdot v) =$
2. The Quotient Rule: $\frac{d}{dx}\left(\frac{u}{v}\right) =$
3. Other Trig Functions:
   a) $\frac{d}{dx}(\tan x) =$
   b) $\frac{d}{dx}(\cot x) =$
   c) $\frac{d}{dx}(\csc x) =$
   d) $\frac{d}{dx}(\sec x) =$

B. Examples

1. $y = (x^2 + 1)(x^3 - 3)$
2. $f(x) = \frac{x^2 - 1}{x^2 + 1}$
3. $f(x) = 3x + \cot x$
4. $y = \frac{2x}{\sin x}$

II. Higher-Order Derivatives

A. Find the higher-order Derivative

1. y'' if $y = x^5 - 2x^2 + 3$
2. y''' if $y = x^8 - 2x^2 - 3x + 1$
3. $y^{(4)}$ if $y = x^7 - 2x + 1$

B. Rates of Rates

Acceleration is the derivative of velocity.

Homework:

p.126 – 5, 8, 11, 14, 16, 18, 19, 23, 26, 30, 35, 41, 49, 51, 53, 59, 62, 65, 67, 81, 82, 85, 93, 97, 99, 105-108 all, 111-114 all, 120, 131-137 all

Section 2.4 – The Chain Rule

1. To differentiate composite function using the Chain Rule

A. Composition Functions – Occurs when one function is plugged into another function

$f(x) = (g \circ h)(x) = \sqrt{x^2 - 1}$

$h(x) =$

$g(x) =$

$f(x) = (g \circ h)(x) = \sin(x - 3)$

$h(x) =$

$g(x) =$

$f(x) = (g \circ h)(x) = \tan^2 x$

$h(x) =$

$g(x) =$

B. Derivatives of Composite Functions – The Chain Rule

1. Method 1: $\frac{dy}{dx} =$

a) Recognize *u* (always choose the inner-most expression, usually the part inside brackets, or under the square root sign).
b) Then we need to re-express *y* in terms of *u*.
c) Then we differentiate *y* (with respect to *u*), then we re-express everything in terms of *x*.
d) The next step is to find *du/dx*.
e) Then we multiply *dy/du* and *du/dx*

Examples: Find $\frac{dy}{dx}$ if …

1. $y = (x^2 + 3)^5$
2. $y = \sqrt{4x^2 - x}$
3. $y = \sin(2x^3 - 1)$

2. Method 2: $\frac{d}{dx}[f(u)] =$

a) Identify the outer function and take the derivative of just the outer function.
b) Move to the inner function, take its derivative, and multiply by the outer.
c) Continue process until inner most function is completed.

Examples: Find $f'(x)$ if …

1. $f(x) = \sin(2x^3 - 1)$
2. $f(x) = \sin(1 + \tan 2x)$

C. General Power Rule

Rule: $\frac{d}{dx}(u^n) =$

Examples

1. $y = (3x - 2x^2)^3$

2. $y = (2x + 1)^{-3}$

3. $f(x) = \cos^2 3x$

4. $y = \sqrt[3]{(x^2 - 1)^2}$

Homework:
p.137 – 1-6 all, 7, 11, 15, 20, 23, 25, 29, 46, 47, 51, 59, 65, 67, 70, 73, 77, 81, 83, 86, 89, 91, 92, 95, 107, 108-110 all, 111, 112, 120, 130-132 all

Section 2.5 – Implicit Differentiation

Goals:
1. To find the derivative using implicit differentiation

Notes:

I. Implicit and Explicit Functions

A. Explicit – the variable *y* is explicitly written as a function of *x*.

$y = 3x^2 - 5$

B. Implicit

1. $x^2 + y^2 = 16$
2. $2y = x^2 + \sin y$

II. Implicit Differentiation

A. Differentiating with respect to *x*.

1. $\frac{d}{dx}\left[x^3\right] =$

2. $\frac{d}{dx}\left[y^3\right] =$

3. $\frac{d}{dx}\left[x + 3y\right] =$

4. $\frac{d}{dx}\left[xy^2\right] =$

B. Guidelines

1. Differentiate _____ sides of the equation *with respect to x.*
2. Collect all terms involving _____ on the _____ side of the equation and move all other terms to the _____ side of the equation.
3. Factor _____ out of the left side of the equation.
4. Solve for _____.

C. Examples

1. Find $\frac{dy}{dx}$ if $x^2 - xy + y^2 = 7$

2. Find the tangent line and normal lines to the curve $x^2 - xy + y^2 = 7$ at (-1,2)

3. Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = y$

Homework:
p.146 – 1, 5, 9, 11, 15, 21, 26, 27, 45, 65, 67, 74

Section 2.6 – Related Rates

Goals:
1. To find related rates using implicit differentiation.
2. To solve application problems involving related rates and implicit differentiation.

Notes:

I. Finding related rates (i.e., finding derivatives with respect to time)

Examples

1. $x^2 + y^2 = 16$

2. $V = x^2 y$

3. $y = r \sin\theta$

II. Problem solving with related rates

Examples

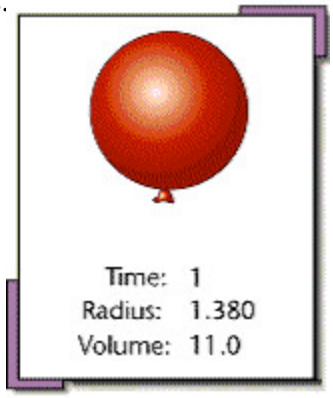
1. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

Equation(s) Needed:

Given Rate(s):

Find:

2. Air is being pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find the rate of change of the radius when the radius is 2 inches.

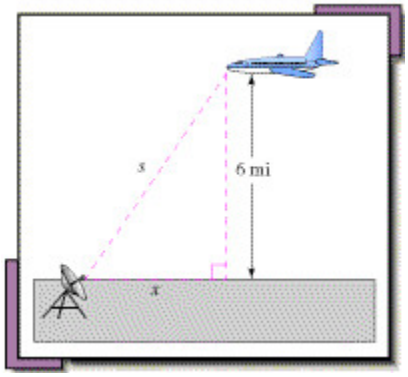


Equation(s) Needed:

Given Rate(s):

Find:

3. An airplane is flying on a flight path that will take it directly over a radar tracking station. If s is decreasing at a rate of 400 miles per hour when $s = 10$, what is the speed of the plane?

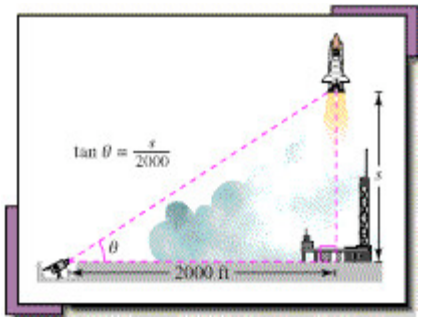


Equation(s) Needed:

Given Rate(s):

Find:

4. Find the rate of change in the angle of elevation of the camera shown in the figure at 10 seconds after lift-off. The space shuttle is rising vertically according to the position equation $s = 50t^2$.



Equation(s) Needed:

Given Rate(s):

Find:

Homework:
p.154 – 1, 3, 4, 5, 8, 11, 12, 14, 16, 17, 22, 24, 25, 27, 29, 31, 33, 38, 39, 44, 46, 50 and p. 139 – 115