

## Chapter Two Massive Review

The following problem is the property of ETS/Collegeboard

### 2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the curve given by  $y^2 = 2 + xy$ .
- (a) Show that  $\frac{dy}{dx} = \frac{y}{2y - x}$ .
- (b) Find all points  $(x, y)$  on the curve where the line tangent to the curve has slope  $\frac{1}{2}$ .
- (c) Show that there are no points  $(x, y)$  on the curve where the line tangent to the curve is horizontal.
- (d) Let  $x$  and  $y$  be functions of time  $t$  that are related by the equation  $y^2 = 2 + xy$ . At time  $t = 5$ , the value of  $y$  is 3 and  $\frac{dy}{dt} = 6$ . Find the value of  $\frac{dx}{dt}$  at time  $t = 5$ .

From: [http://online.math.uh.edu/apcalculus/exams/AP\\_AB\\_version1\\_1.htm](http://online.math.uh.edu/apcalculus/exams/AP_AB_version1_1.htm)

5) Given that

$$5x^3 - 4xy - 2y^2 = 1$$

Determine the change in  $y$  with respect to  $x$ .

- a)  $-\frac{15x^2 - 4}{-4 - 4y}$
- b)  $-\frac{15x^2 - 4y}{-4 - 4y}$
- c)  $-\frac{15x^2 - 4}{-4x - 4y}$
- d)  $-\frac{10x - 4y}{-4x - 2}$
- e)  $-\frac{15x^2 - 4y}{-4x - 4y}$

If  $y^2 - 2xy = 21$ , then what is the value of  $\frac{dy}{dx}$  at the point  $(2, -3)$ ?

If  $\frac{dy}{dx} = \frac{x^2y + y^2x}{3x + y}$ , then where will the curve have horizontal tangents?

What is  $\lim_{h \rightarrow 0} \frac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h}$ ?

If  $f(x) = x^3\sqrt{x}$ , then what is  $f'(x)$ ?

What is the equation of the line tangent to the graph of  $f(x) = 7x - x^2$  at the point where  $f'(x) = 3$ ?

Here's a good one:

The volume of an expanding sphere is increasing at a rate of 12 cubic feet per second. When the volume of the sphere is  $36\pi$  cubic feet, how fast, in square feet per second, is the surface area

increasing? Note:  $V = \frac{4\pi}{3}r^3$  and  $S = 4\pi r^2$

- (A) 8
- (B) 6
- (C)  $8\pi$
- (D)  $\frac{8\pi}{3}$
- (E) 10

If  $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x + 1, & x > 1 \end{cases}$  then  $f'(1)$  is

- (A)  $\frac{1}{2}$
- (B) 1
- (C) 2
- (D) 3
- (E) does not exist

What is  $\lim_{h \rightarrow 0} \frac{\tan(2(x+h)) - \tan(2x)}{h}$ ?

**Some Calculator Problems:**

Let  $f$  be a function given by  $f(x) = \tan x$  and let  $g$  be the function given by  $g(x) = x^2$ . At what value of  $x$  in the interval  $[0, \pi]$  do the graphs of  $f$  and  $g$  have parallel tangent lines?

- (A) 0
- (B) 0.660
- (C) 2.083
- (D) 2.194
- (E) 2.207

Let  $f$  and  $g$  be differentiable functions such that

$$f(1) = 4, g(1) = 3, f'(3) = -5$$

$$f'(1) = -4, g'(1) = -3, g'(3) = 2$$

If  $h(x) = f(g(x))$ , then  $h'(1) =$

- (A) -9
- (B) 15
- (C) 0
- (D) -5
- (E) -12

At the point of intersection of  $f(x) = \cos x$  and  $g(x) = 1 - x^2$ , the tangent lines are

- (A) the same line
- (B) parallel lines
- (C) perpendicular lines
- (D) intersecting but not perpendicular lines
- (E) none of the above

A particle moves along the  $x$ -axis so that its position at any time  $t > 0$  is given by  $x(t) = t^3 + 22t + 3 - 6\cos(\pi t)$ . For what value of  $t$  is the velocity negative?

- (A)  $t = \frac{1}{2}$
- (B)  $t = 1$
- (C)  $t = \frac{3}{2}$
- (D)  $t = 2$
- (E) the velocity is never negative