m

The important algebraic stuff that I missed when I was absent!

Slope in terms of a function: $m = \frac{\Delta y}{\Delta x}$

Let $y = f(x)$ be a linear function. Then the slope, m, between any two points on the line $(a, f(a))$ and $(b, f(b))$, can be written as:

$$m = \frac{f(a) - f(b)}{a - b} \quad \text{OR} \quad m = \frac{f(b) - f(a)}{b - a}$$

Between the two points: $(3, f(3))$ and $(h, f(h))$

$$m = \frac{f(3) - f(h)}{3 - h} \quad \text{OR} \quad m \; \frac{f(h) - f(3)}{h - 3}$$

Between the two points: $(x, f(x))$ and $(x+h, f(x+h))$ ✱✱

$$m = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

Solving equations

Step one: do not panic
Step two: show all steps
Step three: isolate the variable that you are seeking using algebraic properties

$e^{\cos x} = \ln(x+4)$
TI FRIENDLY

Solve for y:
$xz + y = 1 + z$
$xz - xz + y = 1 + z - xz$
$y = 1 + z - xz$

Solve for x:
$xz + y = 1 + z$
$xz + y - y = 1 + z - y$
$\frac{xz}{z} = \frac{1 + z - y}{z}$
$x = \frac{1 + z - y}{z}$

Solve for z:
$xz + y = 1 + z$
$xz - z + y = 1 + z - z$
$xz - z + y - y = 1 - y$
$xz - z = 1 - y$
$\frac{z(x-1)}{x-1} = \frac{1-y}{x-1}$
$z = \frac{1-y}{x-1}$

Solve for y': y' is pronounced "y prime"

JUST REPLACE Z WITH y'

$x\,y' + y = 1 + y'$

$$y' = \frac{1-y}{x-1}$$

good to recycle

Algebra I stuff I must have been absent for:

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$ DIFFERENCE OF 2 SQUARES

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
You can also think of Pascal's triangle when expanding a binomial.

let $a = x$; $b = 2$

use:

$a^3 + 3a^2b + 3ab^2 + b^3$

Expand: $(x+2)^3$

$(x+2)^3$

$= x^3 + 3x^2(2) + 3x(2^2) + 2^3$

$= x^3 + 6x^2 + 12x + 8$

Expand: $(x-2)^3$

$(x-2)^3 = x^3 - 6x^2 + 12x - 8$

Expand: $(3x+2)^3$ let $a = 3x$; $b = 2$

$(3x+2)^3 = (3x)^3 + 3(3x)^2(2) + 3(3x)(2^2) + 2^3$

$= 27x^3 + 54x^2 + 36x + 8$

Expand: $(2x-3)^3$ let $a = 2x$; $b = -3$

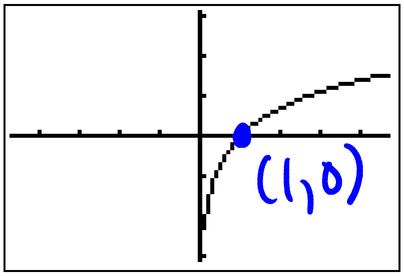
$(2x-3)^3$

$= 8x^3 - 36x^2 + 54x - 27$

Exponential and Logarithmic Functions

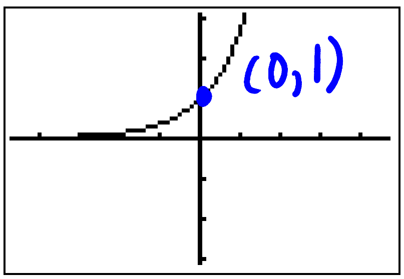
Please learn [or re-learn] these properties and do NOT make up your own rules!

INVERSES



DOMAIN: $(0, \infty)$
RANGE: $(-\infty, \infty)$

This is the graph of $y = \ln x$



DOMAIN: $(-\infty, \infty)$
RANGE: $(0, \infty)$

This is the graph of $y = e^x$

Stuff I need to memorize!

$\ln e = 1$

$\ln 1 = 0$

$\ln a^b = b \ln a$ $\qquad \ln 8^{19} = 19 \ln 8$

$\ln e^x = x \ln e = x$ ☆☆

$e^{\ln x} = x$ $\qquad e^{\ln 819} = 819$

Since $y = \ln x$ is the inverse of $y = e^x$, then we can solve equations using this relationship.

Solve: $\ln(7x) = 11$

To solve for x we can exponentiate both sides of the equation.

$$e^{\ln(7x)} = e^{11}$$

$$7x = e^{11}$$

$$x = \frac{e^{11}}{7}$$

use e as a base

Solve: $e^{7x} = 11$

To solve for x we can take the $\ln$ of both sides of the equation.

$$\ln\left(e^{7x}\right) = \ln(11)$$

$$7x = \ln(11)$$

$$x = \frac{\ln(11)}{7}$$

Remember – all the rules of exponents also apply to the function $y = e^x$

Note: It pays to simplify if you can!

Solve the following:

1. $\ln\left(e^{5x}\right) = 15$

$5x = 15$

$x = 3$

2. $\dfrac{e^{x+5}}{e^{5}} = 3$

WHEN DIVIDING TERMS WITH LIKE BASES, SUBTRACT EXPONENTS

$e^{x} = 3$

$\ln e^{x} = \ln 3$

$x = \ln 3$

3. $\left(e^{3}\right)^{2x}=\left(e^{3}\right)\left(e^{2x}\right)$

$e^{6x} = e^{3+2x}$

$6x = 3+2x$

$4x = 3$

$x = \frac{3}{4}$

4. Solve for y: $e^{y}=x^{3}-15$

$\ln e^{y} = \ln(x^3-15)$

$y = \ln(x^3-15)$

Domain: $x^3-15>0$

$x^3>15$

$x>\sqrt[3]{15}$

5. Solve for y: $\frac{1}{2}e^{2y}=x^{3}+e$

$e^{2y} = 2x^3+2e$

$\ln(e^{2y}) = \ln(2x^3+2e)$

$2y = \ln(2x^3+2e)$

$y = \frac{1}{2}\ln(2x^3+2e)$

$y = \ln\sqrt{2x^3+2e}$

Domain: $2x^3+2e>0$

$x^3>-e$

$x>\sqrt[3]{-e}$

6. Solve for x: $A = Pe^{rx}$

$$\frac{A}{P} = \frac{Pe^{rx}}{P}$$

$$\frac{A}{P} = e^{rx}$$

$$\ln\left(\frac{A}{P}\right) = \ln e^{rx}$$

$$\frac{\ln\left(\frac{A}{P}\right)}{r} = \frac{rx}{r}$$

$$\frac{\ln\left(\frac{A}{P}\right)}{r} = x$$

FANCY-PANTS ANSWER

$$\frac{\ln A - \ln P}{r} = x$$

IF needed, READ chapter P.