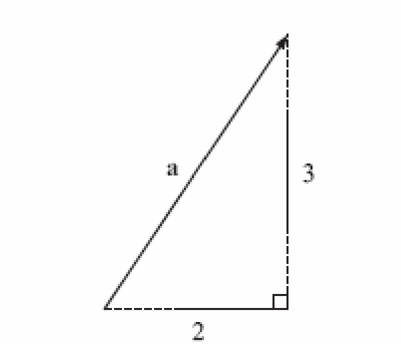
IBSL VECTOR DAY 3

Length of a vector

The length of a vector is called its **magnitude** and can be easily found using the distance formula.



Let's consider vector **a**

It can be represented by the column vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

You can just use the distance formula [based on the Pythagorean Theorem]

$$
\begin{aligned}
|\,a\,| &= \sqrt{2^2 + 3^2} \\
&= \sqrt{13}
\end{aligned}
$$

In general, if **a** $= \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, then $|\,a\,| = \sqrt{(a_1)^2 + (a_2)^2}$

Let's consider Example 15 on page 364

Example 15

If $\mathbf{p} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ find: **a** $|\mathbf{p}|$ **b** $|\mathbf{q}|$ **c** $|\mathbf{p} - 2\mathbf{q}|$

a $\mathbf{p} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ $\therefore$ $|\mathbf{p}| = \sqrt{9+25} = \sqrt{34}$ units

b $\mathbf{q} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $\therefore$ $|\mathbf{q}| = \sqrt{1+4} = \sqrt{5}$ units

c $\mathbf{p} - 2\mathbf{q} = \begin{bmatrix} 3 \\ -5 \end{bmatrix} - 2\begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ $\therefore$ $|\mathbf{p} - 2\mathbf{q}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$ units

Solving vector equations in 2-d

Note: We can not divide a vector by a scalar but we can multiply a vector by a scalar.

Let **x** and **a** be vectors such that $2x = a$, then $x = \frac{1}{2}a$ good

Do not write $\frac{a}{2}$ NOT good

Some handy-dandy rules for solving vector equations:

If $x + a = b$, then $x = b - a$.

If $kx = a$, then $x = \frac{1}{k}a$

JUST LIKE MATRICES

Let's consider Example 16 on page 365

Example 16

Solve for x: **a** $3x - r = s$ **b** $c - 2x = d$

a
$$3x - r = s$$
$$\therefore \quad 3x = s + r$$
$$\therefore \quad x = \tfrac{1}{3}(s + r)$$
DO NOT DIVIDE!

b
$$c - 2x = d$$
$$\therefore \quad c - d = 2x$$
$$\therefore \quad \tfrac{1}{2}(c - d) = x$$
DO NOT DIVIDE!

Try on your own, page 365 #1 [Solve for x]

(a) $2x = q$

$x = \frac{1}{2} q$

(b) $\frac{1}{2}x = n$

$x = 2n$

(c) $-3x = p$

$x = -\frac{1}{3} p$

(d) $q + 2x = r$

$x = \frac{1}{2}(r - q)$

(e) $4x - 5x = t$

$-x = t$

$x = -t$

(f) $4m - \frac{1}{3}x = n$

$x = -3(n - 4m)$

$x = -3n + 12m$

Vectors in coordinate geometry

We should be able to write a vector between any two points and then find the magnitude of that vector.

$\vec{OA} = \begin{pmatrix} x_A \\ y_A \end{pmatrix}$

$\vec{AB} = \vec{AO} + \vec{OB}$

Let Point A be the point (x_A, y_A)

Let Point B be the point $\left(x_B, y_B\right)$ $\vec{OB}$ $\begin{pmatrix} x_B \\ y_B \end{pmatrix}$

The vector $\vec{AB} = \begin{bmatrix} x_B - x_A \\ y_B - y_A \end{bmatrix}$

Which is the change in the x-values and the changes in the y-values.

Let's consider Example 17 on page 366

$\vec{PQ} = \vec{PO} + \vec{OQ}$

$= \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

Example 17

If P is $(-1, 2)$ and $Q(3, 1)$ find

a $\vec{PQ}$ **b** the distance from P to Q.

a $\vec{PQ}$

$= \begin{bmatrix} 3-(-1) \\ 1-2 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

b distance

$= |\vec{PQ}|$

$= \sqrt{4^2 + (-1)^2}$

$= \sqrt{17}$ units

Notice that if $\vec{AB} = \begin{bmatrix} x_B - x_A \\ y_B - y_A \end{bmatrix}$ then $\vec{BA} = \begin{bmatrix} x_A - x_B \\ y_A - y_B \end{bmatrix}$.

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Vector Equality

Two vectors are equal if they have the same length and direction. Namely,

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} r \\ s \end{bmatrix} \Leftrightarrow p = r \quad and \quad q = s$$

Try on your own [silently] page 367 #1

$A(3, -2)$, $B(2, 6)$, $C(-1, -4)$

$\vec{AB} = \vec{AO} + \vec{OB}$

$= \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 8 \end{pmatrix}$

$|\vec{AB}| = \sqrt{(-1)^2 + 8^2}$

$= \sqrt{65}$

$\vec{BC} = \vec{BO} + \vec{OC}$

$= \begin{pmatrix} -2 \\ -6 \end{pmatrix} + \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ -10 \end{pmatrix}$

$|\vec{BC}| = \sqrt{(-3)^2 + (-10)^2}$

$= \sqrt{109}$

$$\vec{CA} = \vec{CO} + \vec{OA}$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$|\vec{CA}| = \sqrt{4^2+2^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

Homework: pages 376 and 377 Review Set 15B
#2, 3, 4, 5, 6, 7