Circular functions and trigonometry

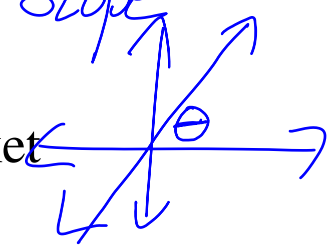
3.1 The circle: radian measure of angles; length of an arc; area of a sector [these formulas are in the formula packet]

3.2 Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle.

Definition of $\tan\theta$ as $\dfrac{\sin\theta}{\cos\theta}$

GRADIENT MEANS SLOPE

[Lines through the origin can be expressed as $y = x\tan\theta$, with gradient $\tan\theta$.]



3.3 The identity $\sin^2\theta + \cos^2\theta = 1$

Double-angle formulas –given in formula packet

3.4 The graphs of $\sin x$, $\cos x$, *and* $\tan x$

Functions of the form: $f(x) = a\sin(b(x+c)) + d$

3.5 Solving trigonometric equations

3.6 Solutions of triangles [Law of Cosines, Law of Sines, and finding the area of a triangle using trigonometry – these formulas are given in the formula packet]

Since you all have had trigonometry in some previous class, you can simply read the trig chapters in our somewhat excellent textbook or go to: http://ibmaths.com/slnotes/trig.pdf to see the revision notes.

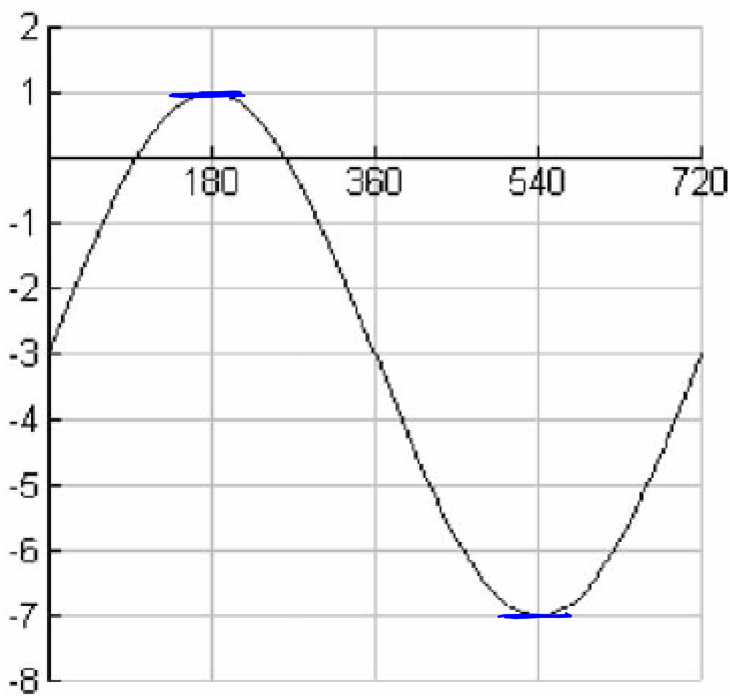
Let's consider some IB-type problems and to see how much you need to review on your own.

From: http://ibmaths.com [non-calculator]

Paper D IB SL Paper 1 Practice Papers

5. *[Maximum mark 4]*

The graph below shows the curve of $y = 4\sin(\frac{1}{2}x) - p$.



degrees are implied
non-calculator

a) Find the amplitude of the function.

b) Find the period of the function.

c) Find the value of p.

Amp = 4
period = 720°
p = 3

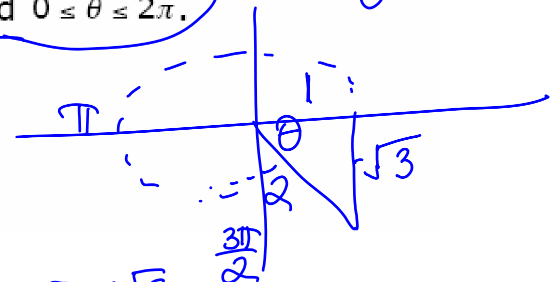
4. *[Maximum mark 6]*

NON-CALC

$\theta = 300^\circ$

Given that $\sin\theta = -\dfrac{\sqrt{3}}{2}$ and $\cos\theta = \dfrac{1}{2}$, and $0 \le \theta \le 2\pi$.

a) find the value of θ. $\theta = \frac{5\pi}{3}$



b) Find the exact value of $\tan\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

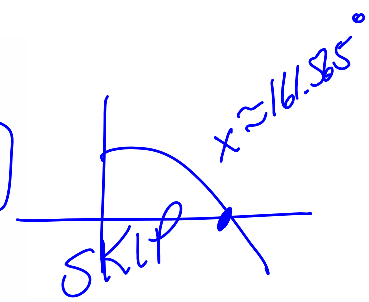
Now for a few GDC problems:

NON-CALCULATOR

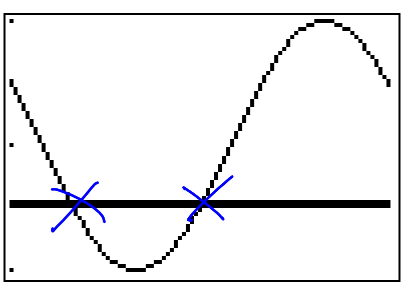
1. *[Maximum 4 marks]*

Solve the following equations for $\boxed{0^\circ \le x \le 180^\circ}$.

a) $3\sin\theta + \cos\theta = 0$ $\theta \approx 161.565^\circ$

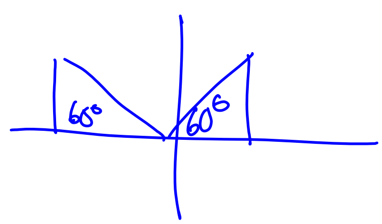


b) $4\cos^2\theta - 1 = 0$



$\theta = 60^\circ$

$\theta = 120^\circ$



$$4\cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \pm\frac{1}{2}$$

I put together the important parts of the sine function from chapter 13 in our textbook.
Note: Our textbook is Mathematics for the International student from Haese and Harris publications [but not officially endorsed by IBO]

Here is it:

- in $y = A\sin x$, A affects the amplitude and the amplitude is $|A|$
- in $y = \sin Bx$, $B > 0$, B affects the period and the period is $\frac{2\pi}{B}$. OR $\frac{360^\circ}{B}$

- $y = \sin(x - C)$ is a **horizontal translation** of $y = \sin x$ through C units.
- $y = \sin x + D$ is a **vertical translation** of $y = \sin x$ through D units.
- $y = \sin(x - C) + D$ is a **translation** of $y = \sin x$ through vector $\begin{bmatrix} C \\ D \end{bmatrix}$. ☆☆☆

So, what does the graph of $y = 3\sin(4x) - 5$ look like?

AMP = 3

PERIOD = $\frac{2\pi}{4} = \frac{\pi}{2}$

VERTICAL TRANSLATION 5 DOWN

$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$ TRANSLATION

What does the graph of $y = 3\sin(x + 4) + 5$ look like [in comparison to the graph above?

AMP = 3

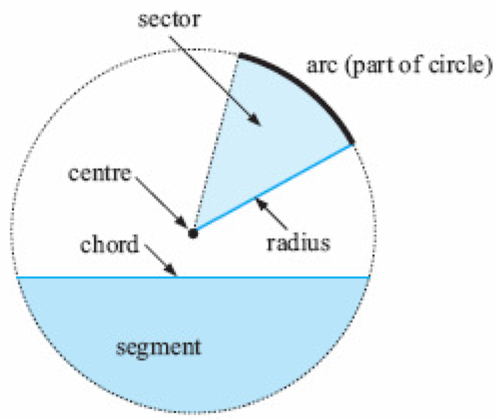
PERIOD = 2π

TRANSLATION

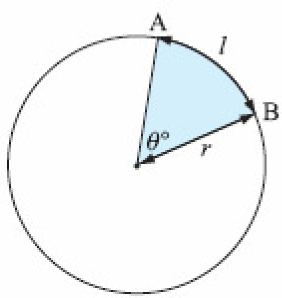
$\begin{bmatrix} -4 \\ 5 \end{bmatrix}$ HORIZONTAL VERTICAL

How comfortable are you with areas and perimeters of sectors? How about the Laws of Sine and Cosine? Here are highlights from chapter 12 of our textbook:

Reminder:



SECTORS



Consider a sector of a circle of radius r and angle θ^o at the centre.

If l is the length of the arc from A to B then:

- $l = \left(\frac{\theta}{360}\right) \times 2\pi r$
- area $= \left(\frac{\theta}{360}\right) \times \pi r^2$

Above graphic from page 239

The Cosine Rule from page 241:

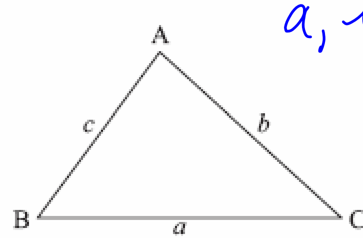
A, B, C are angles

a, b, c are lengths of SIDES

The **cosine rule** involves the sides and angles of a triangle.

In any ΔABC:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{or} \quad b^2 = a^2 + c^2 - 2ac\cos B$$
$$\text{or} \quad c^2 = a^2 + b^2 - 2ab\cos C$$



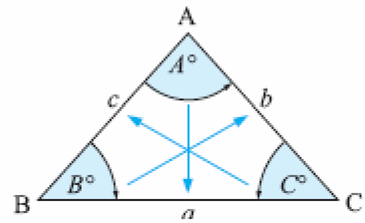
From page 241: RE-ARRANGEMENT OF FORMULA

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

A formula that you might be familiar with:

LABELLING TRIANGLES

If triangle ABC has angles of size $A°$, $B°$, $C°$, the sides opposite these angles are labelled a, b and c respectively.



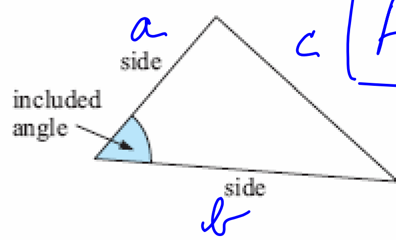
area $= \frac{1}{2}ab\sin C$.

[From page 236 and it is in the formula packet]

Summary:

Given the lengths of two sides of a triangle and the angle between them (called the **included angle**), the area of the triangle is

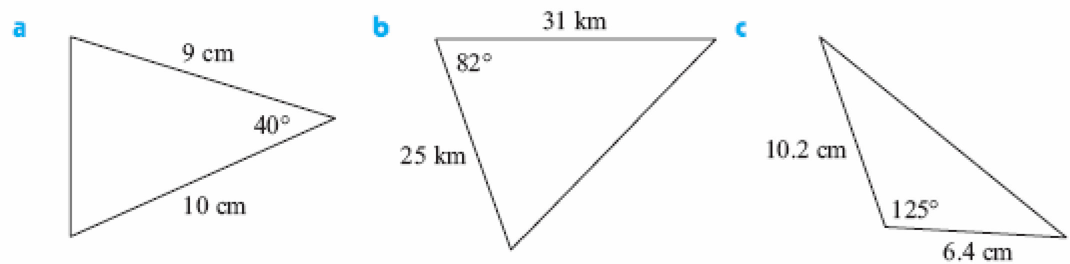
a half of the product of two sides and the **sine of the included angle**.



c $\boxed{A = \frac{1}{2}ab \sin C}$

Let's try a few problems:

1 Find the area of:



1a

$a = \frac{1}{2} ab \sin C$

$a = \frac{1}{2}(9)(10) \sin 40°$

$a \approx 28.905 \text{ cm}^2$

1b

$a = \frac{1}{2} ab \sin C$

$a = \frac{1}{2}(31)(25) \sin 82°$

$a = 383.729 \text{ km}^2$

1c

$a = \frac{1}{2} ab \sin C$

$a = \frac{1}{2}(10.2)(6.4) \sin 125^\circ$

$a = 26.737 \text{ cm}^2$

Here's a sector example from page 239:

Example 3

A sector has radius 12 cm and angle 65°. Find:

a its arc length **b** its area

a arc length $= \left(\frac{\theta}{360}\right) \times 2\pi r$

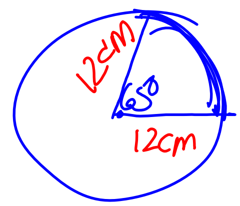
$= \frac{65}{360} \times 2 \times \pi \times 12$

$\doteqdot 13.6$ cm $\approx$

b area $= \left(\frac{\theta}{360}\right) \times \pi r^2$

$= \frac{65}{360} \times \pi \times 12^2$

$\doteqdot 81.7 \text{ cm}^2$



Now you try one:

2 A sector has an angle of 107.9° and an arc length of 5.92 m. Find:

a its radius **b** its area.

$l = \frac{\theta}{360}(2\pi r)$

$5.92 = \frac{107.9^\circ}{360}(2\pi r)$

$3.144 \text{ m} \approx r$

$a = \frac{\theta}{360}(\pi r^2)$

$a = \frac{107.9}{360}(\pi (3.144)^2)$

$a \approx 9.308 \text{ m}^2$

Example 4

A sector has radius 8.2 cm and arc length 13.3 cm. Find its angle.

$$\text{arc length} = \left(\frac{\theta}{360}\right) \times 2\pi r$$

$$\therefore \quad \frac{\theta}{360} \times 2 \times \pi \times 8.2 = 13.3$$

$$\therefore \quad \theta \times 2 \times \pi \times 8.2 = 360 \times 13.3 \qquad \{\text{multiply both sides by } 360\}$$

$$\therefore \quad \theta = \frac{360 \times 13.3}{(2 \times \pi \times 8.2)} \doteqdot 92.93$$

So, its angle is 92.9°.

Now you try!

4 Find the angle of a sector of:

a radius 4.3 m and arc length 2.95 m

b radius 10 cm and area 30 cm^2.

If time, do page 304 #9 [image from Google]

WATER BEETLE "Ringo"

$P(t) = 5 + 2\sin\left(\frac{\pi t}{3}\right)$ for $[0, 8]$ RADIANS IMPLIED

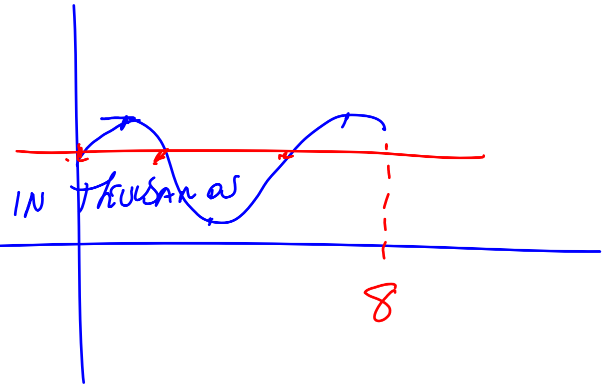
where t is the number of weeks [gdc-friendly]

(a) Initial population?

$P(0) = 5 + 2\sin(0)$

$P(0) = 5$ BUGS [IN THOUSANDS]

(b) Smallest and Largest populations?



MIN VALUE 3 BEETLES IN THOUSANDS

MAX VALUE 7 BEETLES

(c) During what time interval(s) did the population exceed 6000?

FIND INTERSECTIONS

OF $y = 5 + 2\sin\left(\frac{\pi t}{3}\right)$

and $y = 6$

2 INTERVALS

$(.5, 2.5)$ and $(6.5, 8]$

Homework

Read Chapter 12 and do the following problems:

Page 237 #2, 4, 7; page 238 #9; page 240 #4, 5

[You really should read the chapter first.]

Test on Thursday

10/16