Previous Knowledge

See pages 236 and 239 for area of a triangle and area of a sector

IBSL Trigonometry Problems from Previous Exams

From: November 2006 paper 1

13. The function f is defined by $f : x \mapsto 30 \sin 3x \cos 3x$, $0 \le x \le \frac{\pi}{3}$.

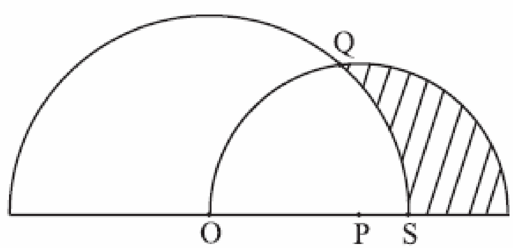
(a) Write down an expression for $f(x)$ in the form $a \sin 6x$, where a is an integer.

(b) Solve $f(x) = 0$, giving your answers in terms of π.

November 2006 Paper 2

5. *[Maximum mark: 17]*

The following diagram shows two semi-circles. The larger one has centre O and radius 4 cm. The smaller one has centre P, radius 3 cm, and passes through O. The line (OP) meets the larger semi-circle at S. The semi-circles intersect at Q.



(a) (i) Explain why OPQ is an isosceles triangle.

(ii) Use the cosine rule to show that $\cos O\hat{P}Q = \frac{1}{9}$.

(iii) Hence show that $\sin \hat{\text{OPQ}} = \frac{\sqrt{80}}{9}$.

(iv) Find the area of the triangle OPQ.

(b) Consider the smaller semi-circle, with centre P.

(i) Write down the size of $\hat{\text{OPQ}}$.

(ii) Calculate the area of the sector OPQ.

(c) Consider the larger semi-circle, with centre O. Calculate the area of the sector QOS.

(d) Hence calculate the area of the shaded region.

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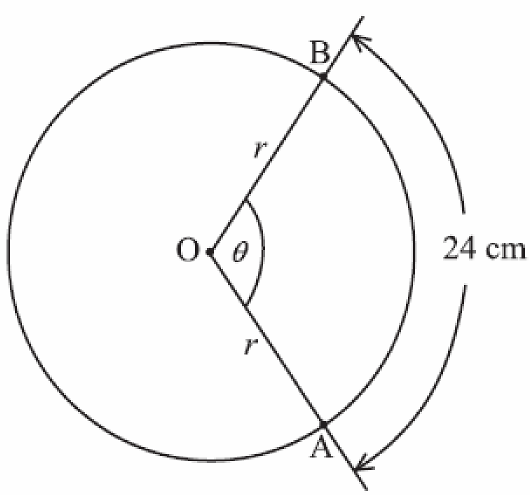
8. The diagram below shows a circle of radius r and centre O. The angle $\text{AOB} = \theta$.

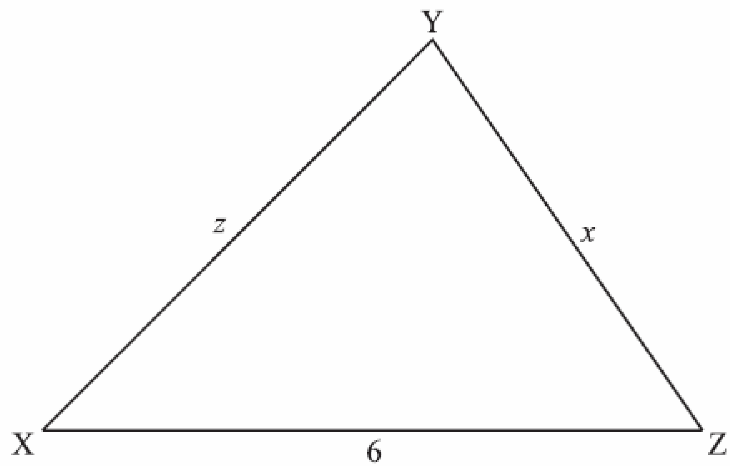


The length of the arc AB is 24 cm. The area of the sector OAB is 180 cm^2.

Find the value of r and of θ.

May 2006 Paper 2

The triangle XYZ has $XZ = 6$, $YZ = x$, $XY = z$ as shown below. The perimeter of triangle XYZ is 16.



(b) (i) Express z in terms of x.

(ii) Using the cosine rule, express z^2 in terms of x and $\cos Z$.

(iii) Hence, show that $\cos Z = \dfrac{5x-16}{3x}$.

Let the area of triangle XYZ be A.

(c) Show that $A^2 = 9x^2 \sin^2 Z$.

(d) Hence, show that $A^2 = -16x^2 + 160x - 256$.

(e) (i) Hence, write down the maximum area for triangle XYZ.

(ii) What type of triangle is the triangle with maximum area?

Solution to #13 November 2006

QUESTION 13

(a) Evidence of choosing the double angle formula *(M1)*

$f(x) = 15\sin(6x)$ *A1* *N2*

(b) Evidence of substituting for $f(x)$ *(M1)*

e.g. $15\sin 6x = 0$, $\sin 3x = 0$ **and** $\cos 3x = 0$

$6x = 0,\ \pi,\ 2\pi$

$x = 0,\ \frac{\pi}{6},\ \frac{\pi}{3}$ *A1A1A1* *N4*

Solution to November 2006 paper 2

QUESTION 5

(a) (i) $\mathrm{OP} = \mathrm{PQ}\ \ (= 3\text{ cm})$

So $\Delta\mathrm{OPQ}$ is isosceles

(ii) Using cos rule correctly *e.g.* $\cos \mathrm{O\hat{P}Q} = \frac{3^2 + 3^2 - 4^2}{2 \times 3 \times 3}$

$\cos \mathrm{O\hat{P}Q} = \frac{9 + 9 - 16}{18} \quad \left(= \frac{2}{18}\right)$

$\cos \mathrm{O\hat{P}Q} = \frac{1}{9}$

(iii) Evidence of using $\sin^2 A + \cos^2 A = 1$

$\sin \mathrm{O\hat{P}Q} = \sqrt{1 - \frac{1}{81}} \quad \left(= \sqrt{\frac{80}{81}}\right)$

$\sin \mathrm{O\hat{P}Q} = \frac{\sqrt{80}}{9}$

(iv) Evidence of using area triangle OPQ $= \frac{1}{2} \times OP \times PQ \sin P$

e.g. $\frac{1}{2} 3 \times 3 \frac{\sqrt{80}}{9}$, $\frac{9}{2} \times 0.9938...$

Area triangle OPQ $= \frac{\sqrt{80}}{2}$ $\left(= \sqrt{20}\right)$ $(= 4.47)$

(b) (i) $O\hat{P}Q = 1.4594...$

$O\hat{P}Q = 1.46$

(ii) Evidence of using formula for area of a sector

e.g. Area sector OPQ $= \frac{1}{2} \times 3^2 \times 1.4594...$

$= 6.57$

$\frac{2\pi}{360°}$

(c) $Q\hat{O}P = \frac{\pi - 1.4594...}{2}$ $(= 0.841)$

Area sector QOS $= \frac{1}{2} \times 4^2 \times 0.841$

$= 6.73$

Question 5 continued

(d) Area of small semicircle is 4.5π $(= 14.137...)$ ***A1***

Evidence of correct approach ***M1***

e.g. Area = area of semicircle − area sector OPQ − area sector QOS + area triangle POQ

Correct expression ***A1***

e.g. $4.5\pi - 6.5675... - 6.7285... + 4.472...$, $4.5\pi - (6.7285... + 2.095...)$,

$4.5\pi - (6.5675... + 2.256...)$

Area of the shaded region = 5.31 ***A1*** ***N1***

[4 marks]

Total [17 marks]

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QUESTION 8

METHOD 1

Evidence of correctly substituting into $l = r\theta$

Evidence of correctly substituting into $A = \frac{1}{2}r^2\theta$

For attempting to solve these equations
eliminating one variable correctly

$r = 15 \quad \theta = 1.6 \quad (= 91.7^\circ)$

METHOD 2

Setting up and equating ratios

$$\frac{24}{2\pi r} = \frac{180}{\pi r^2}$$

Solving gives $r = 15$

$$r\theta = 24 \quad \left(\text{or } \frac{1}{2}r^2\theta = 180\right)$$

$\theta = 1.6 \quad (= 91.7^\circ)$

$r = 15 \quad \theta = 1.6 \quad (= 91.7^\circ)$

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(b)	(i)	$z = 10 - x$ (accept $x + z = 10$)	***A1***	***N1***
	(ii)	$z^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos Z$	***A2***	Page: 6
	(iii)	Substituting for z into the expression in part (ii)	***(M1)***	
		Expanding $100 - 20x + x^2 = x^2 + 36 - 12x\cos Z$	***A1***	
		Simplifying $12x\cos Z = 20x - 64$	***A1***	
		Isolating $\cos Z = \frac{20x - 64}{12x}$	***A1***	
		$\cos Z = \frac{5x - 16}{3x}$	***AG***	***N0***

Note: Expanding, simplifying and isolating may be done in any order, with the final ***A1*** being awarded for an expression that clearly leads to the required answer.

[7 marks]

(c) Evidence of using the formula for area of a triangle $\left(A = \frac{1}{2} \times 6 \times x \times \sin Z\right)$

$A = 3x \sin Z \quad \left(A^2 = \frac{1}{4} \times 36x^2 \sin^2 Z\right)$

$A^2 = 9x^2 \sin^2 Z$

Question 3 continued

(d) Using $\sin^2 Z = 1 - \cos^2 Z$

Substituting $\frac{5x-16}{3x}$ for $\cos Z$

for expanding $\left(\frac{5x-16}{3x}\right)^2$ to $\left(\frac{25x^2 - 160x + 256}{9x^2}\right)$

for simplifying to an expression that clearly leads to the required answer

e.g. $A^2 = 9x^2 - (25x^2 - 160x + 256)$

$A^2 = -16x^2 + 160x - 256$

(e) (i) 144 (is maximum value of A^2, from part (a))

$A_{\max} = 12$

(ii) Isosceles