Geometric Series and Sigma $\sum$ notation

A geometric series is the addition of successive terms of a geometric sequence.

Can we use the formula from the arithmetic series? NO!

Let $\{u_n\}$ be a geometric sequence.
Then, the finite geometric series is:
$S_n = u_1 + u_2 + u_3 + ... + u_n$ which we can rewrite as
$S_n = u_1 + u_1 r + u_1 r^2 + u_1 r^3 + ... + u_1 r^{n-1}$
Through algebraic "magic", we can multiply both sides by r to get:
$r S_n = ru_1 + r^2 u_1 + r^3 u_1 + r^4 u_1 + ... + r^n u_1$
Now let's use some more algebraic "magic" and subtract $S_n - rS_n$ which will equal

$$\left(\underline{u_1} + \cancel{r u_1} + \cancel{r^2 u_1} + \cancel{r^3 u_1} + ... + \cancel{r^{n-1} u_1}\right) - \left(\cancel{r u_1} + \cancel{r^2 u_1} + \cancel{r^3 u_1} + ... + \underline{r^n u_1}\right)$$

Cancel where you can!
Our result it:

$S_n - r S_n = u_1 - r^n u_1$ Now let's factor the left side $a = u_1$

$S_n (1 - r) = u_1 - r^n u_1$ Now isolate S_n

$$S_n = \frac{u_1 - r^n u_1}{1 - r} \quad \text{or} \quad \frac{u_1(1 - r^n)}{1 - r} \quad \text{or} \quad \frac{a(1 - r^n)}{1 - r} \quad [r \neq 1]$$

If $r > 1$, then both the numerator and denominator are negative, and it might be more convenient to use this formula:

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

Let's look at Example 16 on page 54

Example 16

Find the sum of $2 + 6 + 18 + 54 + \ldots$ to 12 terms.

The series is geometric with $u_1 = 2$, $r = 3$ and $n = 12$.

So, $S_{12} = \dfrac{2(3^{12} - 1)}{3 - 1}$ $\quad \left\{\text{Using } S_n = \dfrac{u_1(r^n - 1)}{r - 1}\right\}$

$= \dfrac{\cancel{2}(3^{12} - 1)}{\cancel{2}}$

$= 531\,440$

OR $S_n = \dfrac{u_1(1 - r^n)}{1 - r}$

The Sum of an Infinite Geometric Series

$5 = u_1$

$5 + \frac{1}{5} + \frac{1}{25} + \frac{1}{625} + \ldots$ $\quad r = \frac{1}{5}$

When $|r| < 1$, the terms of a geometric series decrease as n increases. In this case, a series with an infinite number of terms has a finite sum and we can say that as $n \to \infty$, the sum converges. This sum, S_∞, can be found with the following formula:

$$S_\infty = \frac{u_1}{1 - r} \text{ or } \frac{a}{1 - r} \qquad |r| < 1$$

$S_\infty = \dfrac{5}{1 - \frac{1}{5}}$

$S_\infty = \dfrac{25}{4}$

Note: If $|r| > 1$, then the infinite series diverges and has no sum.

$5 + 25 + 125 + 625 + \ldots$

Consider: $S_n = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots + \frac{1}{3^n}$ Find S_n as $n \to \infty$

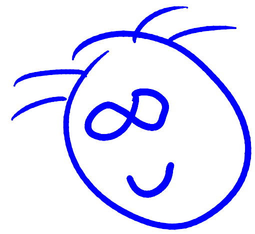
$$S_\infty = \frac{u_1}{1-r} \qquad \text{where } u_1 = \frac{1}{3} \text{ and } r = \frac{1}{3}$$

$$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}}$$

$$S_\infty = \frac{\frac{1}{3}}{\frac{2}{3}}$$

$$S_\infty = \frac{1}{2}$$

Our first infinite sum!



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Sigma notation

Mathematicians like to use "special" notation to indicate operations. The Greek letter sigma, $\sum$, is used to denote a sum.

Just like in Calculus, the $\sum$ will have a lower and an upper bound which tells us where to begin and end the sum.

Examples

$$\sum_{n=1}^{5} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$= 1 + 4 + 9 + 16 + 25$$

$$= 55$$

$$\sum_{k=1}^{5} k^2$$

Is this series arithmetic, geometric or Canadian?

$$\sum_{n=1}^{3} (3n+4) = 7 + 10 + 13$$

$$= \cancel{20}\ 30$$

$d = 3$

$u_1 = 7$

Is this series arithmetic, geometric, or Canadian?

$$\sum_{n=1}^{4} (3)\left(\frac{1}{2}\right)^n = (3)\left(\tfrac{1}{2}\right) + (3)\left(\tfrac{1}{2}\right)^2 + (3)\left(\tfrac{1}{2}\right)^3 + (3)\left(\tfrac{1}{2}\right)^4$$

$$= \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$$

$$= \frac{45}{16}$$

Is this series arithmetic, geometric, or Canadian?

Homework:
Page 54 #1a, 1b
Page 55 #2b, 2c and #4
Page 56 #6
Page 57 #3, 4

Review Problems for Friday’s project:
Page 304 [13D] #3a and 3b
Page 59 [2A] #4, 5
Page 60 [2C] #1, 2, 5, 6, 7
Page 193 #1 and be able to transform a quadratic