Another look at Independent Events

Recall that if A and B are independent events are events, then the occurrence of A does not affect B and vice versa.

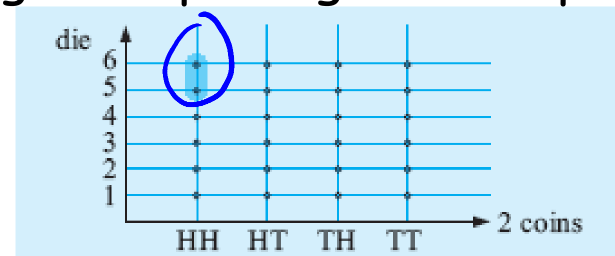
Here are some handy equations to remember:
If A and B are independent events, then

$P(A|B) = P(A)$
$P(B|A) = P(B)$
$P(A \cap B) = P(A)\,P(B)$

We can prove that events are independent based on the above equations. Let's look at Example 26 on page 503.

Two coins are tossed and a die is rolled.
$P(HH) = \frac{1}{4}$
$P(5 \text{ or } 6) = \frac{2}{6} \text{ or } \frac{1}{3}$

To prove that these events are independent events we will consider the following:
Take a look at the grid depicting the sample space



on page 503.

$$P(A)\,P(B) = \frac{1}{12}$$

$$P(A \cap B) = \frac{1}{12} = P(A)\ P(B)$$

Since these are equal, then A and B are independent events. It is possible to have more than two independent events.

Example from Neill and Quadling textbook:
In a game, the player has to toss a coin and then roll a die. The player wins if the coin shows heads and the dice is below 3. Find the probability of winning.

Let W = winning, let H = heads, let N=number on die

P(W)=P(H $\cap$ N = 1 or N = 2)
Are these independent events? [Does the toss of the coin affect the toss of the die?]

Hence, P(W)=P(H) P(N = 1 or N = 2)

$$= \frac{1}{2} \bullet \frac{2}{6}$$

$$= \frac{1}{6}$$

I love Yahtzee! What is the probability of obtaining a "Yahtzee"?

$$P(\text{YAHTZEE}) = 6 \left(\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \right)$$

$$= \frac{6}{6^5} = \frac{1}{6^4} = \frac{1}{1296}$$

1,1,1,1,1
2,2,2,2,2
3,3,3,3,3
4,4,4,4,4
5,5,5,5,5
6,6,6,6,6

Here a different kind of look at independent events:

Example 27

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = p$. Find p if:

a A and B are mutually exclusive

b A and B are independent.

a If A and B are mutually exclusive, $A \cap B = \phi$

and so $P(A \cap B) = 0$

But $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad p = \frac{1}{2} + \frac{1}{3} - 0$

i.e., $p = \frac{5}{6}$

b If A and B are independent, $P(A \cap B) = P(A)\,P(B)$

$= \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$

$\therefore \quad P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$

i.e., $p = \frac{2}{3}$

Homework: page 504 #1-6 all