The Binomial Distribution

We actually did some of these in our probability chapter but this will be a more in-depth look.

Characteristics of a Binomial Experiment

- repetition of a number of independent trials
- only two outcomes [success or failure]
- probability of success is constant for all trials
- there is replacement
- fixed number of trials

Let's consider our textbook's example on pages 721 and 722

Our spinner has 3 blue edges and 1 white edge

P(blue) = $\frac{3}{4}$ and P(white) = $\frac{1}{4}$

We can consider getting a blue as our "success" and getting a white as our "failure"

Notice that there are only two possible outcomes.

In this example, $n = 3$, is the number of times that we will "twirl" the spinner.

X = number of successes [blue]

P(0) will mean that we had no blues or 3 whites

P(1) will mean P(1 blue, 2 whites) We should note that there are 3 ways of obtaining this outcome – the blue could be on the first spin, second spin, or third spin.

P(2) will mean P(2 blue, 1 white) Once again, we should note that there are 3 ways of obtaining this outcome, the white could be on the first spin, second spin, or third spin.

P(3) = P(3 blues)

See the nice tree diagram on page 722.

The notation C_0^3 is equal to $\binom{3}{0}$ or 3 chose 0 – our old friend from Pascal's triangle!

As you can see from the computation on page 722, we can generalize our probability.

$P(X = x)$ where $x = 0,\ 1,\ 2,\ 3$ is equal to

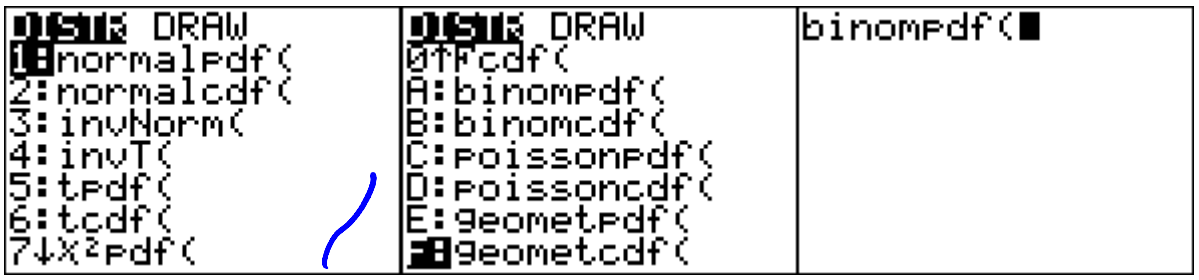
$$P(X = x) = \binom{3}{x}\left(\frac{3}{4}\right)^x\left(\frac{1}{4}\right)^{3-x}$$

$P(X=2) = \binom{3}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^1$

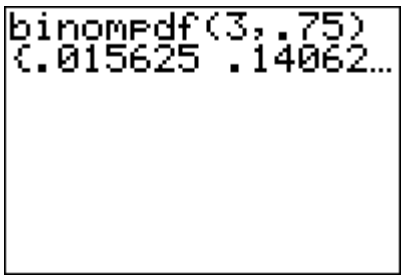
The general case formula is given on page 722.

As luck would have it, our TI can help us out.

Go to the DISTR menu and choose A for the binomial probability distribution function

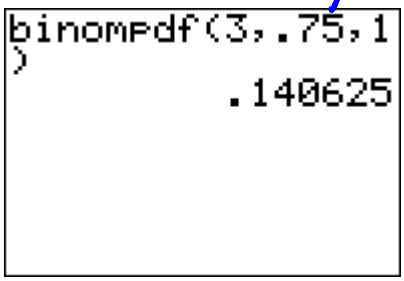


Let's see how this works. We will need to enter n, which is the number of trials, and p, which is the probability of a success.



Cursor to the right and see what numbers you get.

We can also use this TI tool to find just one desired value. For example, if we had 3 twirls, with p =0.75, and we just wanted to know the probability of just one success. In other words, P(X=1). We can do the following:



$$P(X=1) = \binom{3}{1}\left(\frac{3}{4}\right)^{1}\left(\frac{1}{4}\right)^{2}$$

Yes, this is something you should practice so that you are not confused during a test!

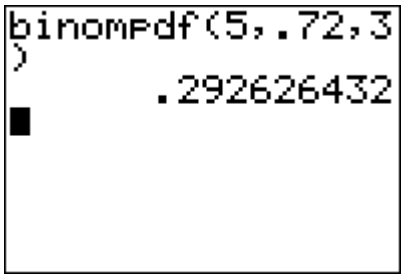
There is another binomial tool on our TI. It is binomcdf and it computes a cumulative probability. Let's see how this works!

See Example 8 on page 723.
72% of union members are in favour of something. A random sample of five members is taken.

X = discrete random variable
$P(X = x)$ is our probability of success where $x = 0,\ 1,\ 2,\ 3,\ 4,\ 5$
$p = 0.72$

$P(X = 3)$ is the probability that 3 of the 5 random union members are in favour.
We can use our TI!

n, p, x=3



$P(X = 3) = \binom{5}{3}(.72)^3(.28)^2$

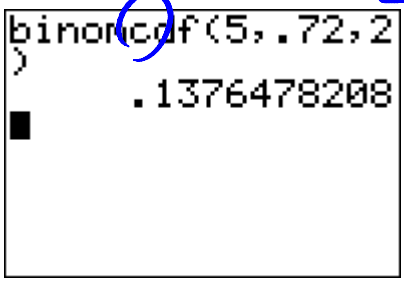
Hence, there is about 29.26% probability of 3 out of the 5 favouring change.

Now to find the probability of at least three members in favour of the change.

$P(X \geq 3) = 1 - P(X \leq 2)$

Let's use our new TI tool – binomcdf – which gives us a cumulative value up to and including 2 members. In other words, it sums up P(X=0)+P(X=1)+P(X=2), thus saving us a lot of work.

n, p, ← P(x≤2) = P(X=0) + P(X=1) + P(X=2)



But we need to subtract this value from 1.

Hence $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - .1376478208$

Or ≈ 0.8623

Let try out our new tricks. Page 723 #2

Family of six children

P(boy)= 0.5

P(girl)=0.5

Let success = boy

P(6 boys)= $\binom{6}{6}(.5)^6(.5)^0$ by HAND

binompdf(6, .5, 6)

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binompdf(6,.5,6)
              .015625
```

$= \frac{1}{64}$ or $\left(\frac{1}{2}\right)^6$

P(2 boys)= $\binom{6}{2}(.5)^2(.5)^4 = \frac{15}{64}$

```
binompdf(6,.5,2)
              .234375
```

P(more than 4 girls) = 1 – $P(G \leq 4)$

```
binomcdf(6,.5,4)
              .890625
```

$\frac{7}{64}$

Hence, P(more than 4 girls = 1 - .890625
= .109375

P(more boys than girls) = P(4 boys)+P(5 boys)+P(6 boys)
OR 1- $P(B \leq 3)$

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binomcdf(6,.5,3)
               .65625
■
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P(more boys)=1-.65625=.34375
$= \frac{11}{32}$

The trick of getting these problems correct is to be a careful reader and knowing how to use your TI.

In order to do this, you need to practice!

Homework: page 723, 724 #3 , 4, 6, 7, 8
Set up, use proper notation and use your TI for the calculations