$r = \begin{pmatrix} x \\ y \end{pmatrix}$

Geometric Applications of $r = a + t\,b$

Let line 1 be: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Let line 2 be: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15 \\ 5 \end{pmatrix} + t\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

Find where these two lines intersect.

In order to intersect, their x-value and y-value must be equal. So let's set them equal!

$$-2 + 3s = 15 - 4t \quad \text{(eq 1)}$$

And

$$1 + 2s = 5 + t \quad \text{(eq 2)}$$

We can easily set up a system of equations with two equations and two unknowns.

ELIMINATION METHOD

$$-2 + 3s = 15 - 4t$$
$$4 + 8s = 20 + 4t$$
$$2 + 11s = 35$$
$$11s = 33$$
$$s = 3$$
$$t = 2$$

$$1 + 2(3) = 5 + t$$
$$2 = t$$

We really should check our solution.

IF $s = 3$ $\quad x = 7 \quad y = 7$

IF $t = 2$ $\quad x = 7 \quad y = 7$ ✓

Hence, our point of intersection is (7, 7)

Let's do #1 on pg 410

Line 1: (AB) $\quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + r\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Line 2: (AC) $\quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Line 3: (BC) $\quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

To graph them accurately, we could convert the lines into Cartesian form and use our g.d.c.

$x = -1 + 3r$

Line 1: $x + 1 = 3r$

$\frac{x+1}{3} = r$

Now let's consider y

$y = 6 - 2r$ Use substitution

WE WANT y IN TERMS of X

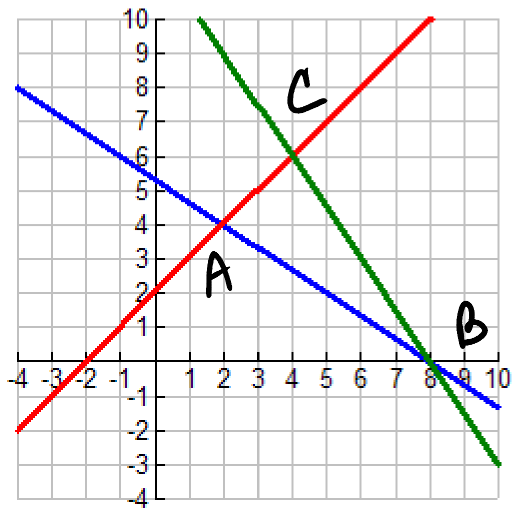
$$y = 6 - 2\left(\frac{x+1}{3}\right)$$ Which we can simplify

$$y = \frac{16}{3} - \frac{2}{3}x$$ AB

Now you find the Cartesian equations for the other two lines.

AC $y = x + 2$

BC $y = -\frac{3}{2}x + 12$



Now we can easily find the points of intersection!

A is $(2, 4)$

B is $(8, 0)$

C is $(4, 6)$

If this is an isosceles triangle, then which sides must be equal?   AB = BC

We need to prove this by finding the length of each side.

A (2, 4);   B (4, 6);   C (8, 0)

We know that

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

Which equals $\begin{pmatrix} -2 \\ -4 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

So, AB = $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{2^2 + 2^2}$

$|\overrightarrow{AB}| = \sqrt{8}$

Now you find the length of the other two sides!

$|\overrightarrow{AC}|$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= \begin{pmatrix} -2 \\ -4 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$|AC| = \sqrt{52}$

$|\overrightarrow{BC}|$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= \begin{pmatrix} -4 \\ -6 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$|BC| = \sqrt{52}$

Now to prove through vector methods:
Line 1 and Line 2 meet at A

line 1 = line 2

$$\begin{pmatrix}-1\\6\end{pmatrix}+r\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}+s\begin{pmatrix}1\\1\end{pmatrix}$$

$-1+3r=0+s$
$6-2r=2+s$

$3r-s=1$
$-2r-s=-4$

$$\begin{pmatrix}3r-s\\-2r-s\end{pmatrix}=\begin{pmatrix}1\\-4\end{pmatrix}$$

$$\begin{pmatrix}3r-s\\-2r-s\end{pmatrix}=\begin{pmatrix}1\\-4\end{pmatrix}$$

Now we just have a system of equations with two equations and two unknowns.

$$\begin{cases}3r-s=1\\-2r-s=-4\end{cases}\qquad r=1,\quad s=2$$

Now to check:
Let $r=1$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-1\\6\end{pmatrix}+\begin{pmatrix}3\\-2\end{pmatrix}$$
$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$$

Let $s=2$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}+2\begin{pmatrix}1\\1\end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

yay!

Now you show that Line 2 and Line 3 meet at C and Line 1 and Line 3 meet at B.

let line 2 = line 3

☆ $\begin{pmatrix} 0 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

let line 1 = line 3

☆☆ $\begin{pmatrix} -1 \\ 6 \end{pmatrix} + r\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 \\ -3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

☆ $\begin{cases} 0 + s = 10 - 2t \\ 2 + s = -3 + 3t \end{cases}$ $\therefore \begin{pmatrix} s + 2t \\ s - 3t \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \end{pmatrix}$

☆☆ $\begin{cases} -1 + 3r = 10 - 2t \\ 6 - 2r = -3 + 3t \end{cases}$ $\therefore \begin{pmatrix} 3r - 2t \\ -2r - 3t \end{pmatrix} = \begin{pmatrix} 11 \\ -9 \end{pmatrix}$

☆ $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ point $s = 4,\ t = 3$

☆☆ $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$ $r = 3,\ t = 1$

yay!

As you can see, these problems take a lot of time.

Homework: page 410 #2, 3 and 4